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DIGITAL COMPUTER ANALYSIS OF RIGID BODY PROBLEMS

CHARLES B. ANTHONY

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RIGID BODY PROBLEMS

by

Charles B. Anthony

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

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the thesis requirements for the degree of

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Abstract.

Subroutine "STADET" analyzes a structure to find if it is stable and determinate. The structure must first be idealized as a rigid body supported by a system of links. The program checks on the arrangement and number of the support links. Flags are set so that this subroutine may be used to select whether to use simple statics or strength of materials methods in subsequent analysis of the problem. Testing of the subroutine is discussed.

Two input-output programs are also described. One calculates the reactions in the equivalent support linkages from the resultant load; the other assumes that the properties of the links are known and calculates the allowable load.

Acknowledgment.

The author wishes to thank Professor Gilles Cantin for his advice and direction. The director and staff of the Computer Center of the United States Naval Postgraduate School were also very helpful and generous with their time.

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Notation.

A	A matrix of coefficients
A, B, C, D	The coefficients of the equation of a plane
$\underline{A}$	A vector
A_x, A_y, A_z	Rectangular components of $\underline{A}$
B	A column matrix
$\underline{B}$	A vector
B_x, B_y, B_z	Rectangular components of $\underline{B}$
$\underline{B}_1$, $\underline{B}_2$, etc.	Vectors made up of the direction cosines of the support links, link cosine vectors
$\underline{B}_i$, $\underline{B}_j$, $\underline{B}_k$	Indicates the ith, jth or kth of $\underline{B}_1$, $\underline{B}_2$, etc.
F	A Fortran floating point, numerical format
$\underline{F}$	A vector force
$\underline{F}_i$	The ith vector force
F_x, F_y, F_z	Rectangular components of $\underline{F}$
F_{xi},F_{yi},F_{zi}	Rectangular components of $\underline{F}_i$
I	A Fortran integer, numerical format
$\underline{i}$, $\underline{j}$, $\underline{k}$	Unit orthogonal base vectors in the x, y, z directions respectively
KPROB	An array used to indicate the kind of problem involved
l, m, n	Direction cosines of a line
$\underline{M}$	Vector moment
M_x, M_y, M_z	Rectangular components of $\underline{M}$
NOSOL	An array used to indicate the reason for the lack of a solution

$\underline{P}_1$, $\underline{P}_2$, etc.	Position vectors giving the locations of the points of application of the links with respect to the first point of application
PLA	An array used to store the coefficients of the equations of the planes determining the lines of action of the links
$\underline{r}$	The position vector of the point of application of a force
r_x, r_y, r_z	Rectangular components of $\underline{r}$
$\underline{u}$	Poynting's vector
W	A weight
x	A column matrix
x, y, z	Rectangular coordinates
x_o, y_o, z_o	Coordinates of a given point
X_1	The scalar result of forming a dot product
$\underline{X}_1$	The vector result of forming a cross product
α	The angle included between two vectors

1. Introduction.

Digital computers have been extensively applied to the solution of mechanics problems. However, these applications have been limited in scope. The computer has been used as a "super slide rule" to solve a particular type of problem which was too time-consuming or too difficult previously. It is the intention of Professor Gilles Cantin to attack the problem of the design of structures by a digital computer in a general rather than a specific manner.

One of the first routines needed for the Cantin Project was a routine to reject unstable structures and, perhaps, make some selection as to the methods of analysis to be used in subsequent tests. The subroutine "STADET" analyzes an idealized structure to find if it is stable and determinate. It sets flags which can be used to print out a description of the kind of problem involved. When the subroutine finds the first condition which would preclude the solution of the problem by simple statics, a flag is set and control returns to the input-output program. If the "error" is caused by a redundant support system, other methods of analysis may be available whereas there is no solution for an "unstable" problem.

The definition of stability which is considered here is more restrictive than the question of stable equilibrium usually encountered in basic statics courses. Here is meant, not the criterion of energy change with small

perturbations, but the geometrical stability of infinitely strong members. As expressed by Kinney,[1]

> A stable structure will support any conceivable system of applied loads, resisting these loads elastically and immediately upon their application the strength of all members and the capacity of all supports being considered infinite.

Consider the two links supporting a weight shown in Fig. 1.

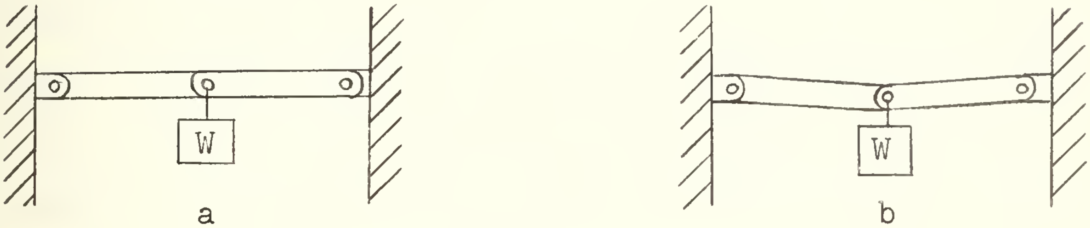

Fig. 1. Support links before and after displacement.

In Fig. 1a, with the links horizontal, there is no vertical component of the tension in the bars to support W. Even with infinitely strong foundations and members, the geometrical arrangement precludes a vertical component until some downward displacement of W has occurred (Fig. 1b). The methods of strength of materials are required to find the equilibrium position of the structure. Since W is not resisted "immediately upon its application," this arrangement fails the test for stability.

Both a structure and its support system must be stable. The problem which is examined here is limited to that of the external stability of a structure. The

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[1] J. Sterling Kinney, Indeterminate Structural Analysis, p. 20, 1957.

structure itself is considered to be a rigid body so that only the system of supporting links is of concern. No loads are imposed during the stability analysis since equilibrium is not involved.

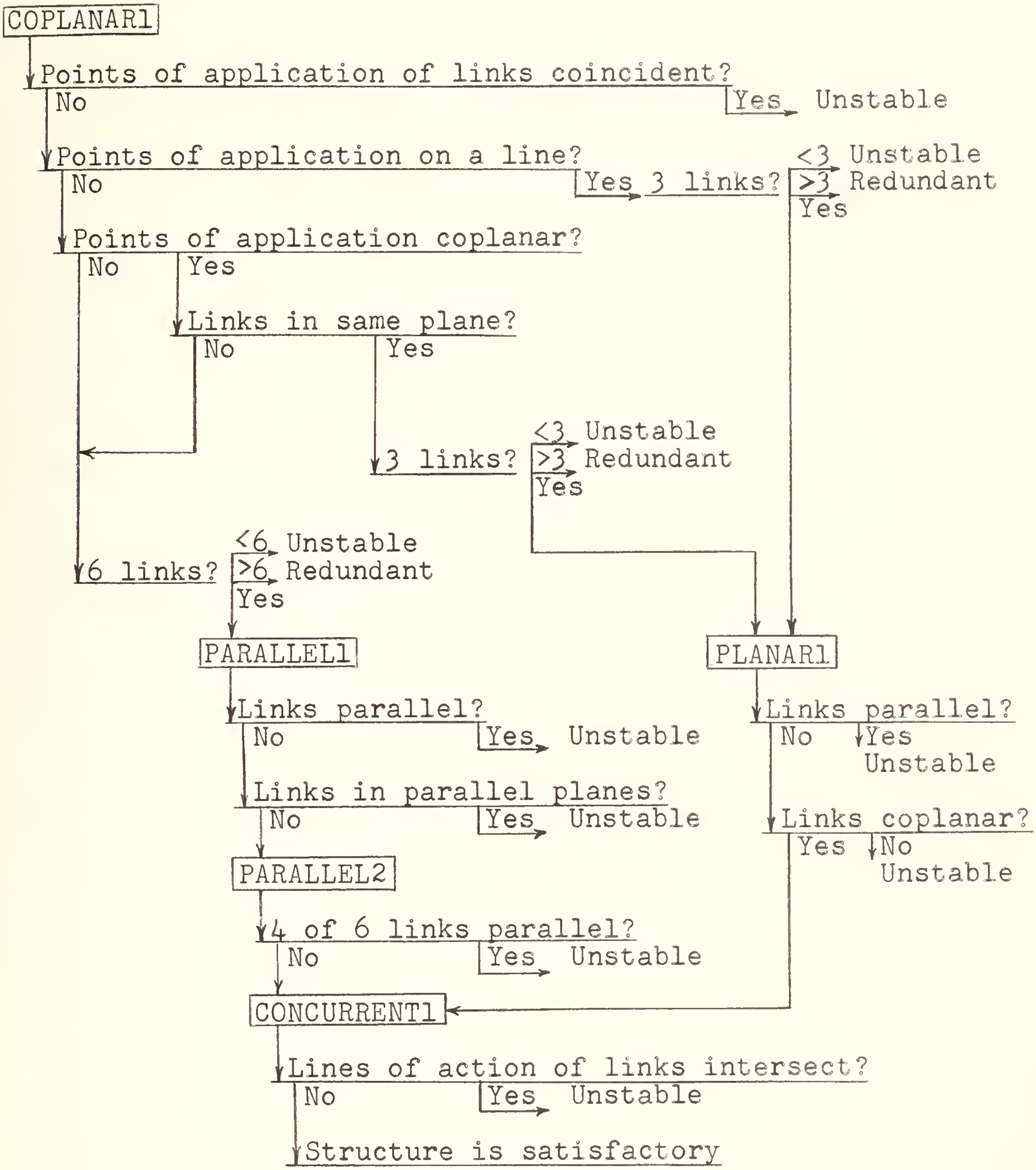
However, in some cases forces such as gravity must be considered in order to have a structure which meets this restricted definition of stability. This is a familiar assumption in basic statics where, for example, the gravity force provides a reaction against upward motion of a roller nest. It is up to the user of this program to decide whether a "gravitational" restraint is sufficient or whether a positive restraint is needed.

The user must first idealize his structure so that the subroutine can operate on it. Appendix A, which shows the linkage diagrams of some common supports, may be helpful in this regard. However, it is at this point that engineering judgment is required. Needless to say, if the proper equivalent support system is not employed, no reliance can be placed on any answers which are produced.

Fig. 2 shows the decisions which are made by the subroutine. The titles within the blocks indicate the basic sections of the subroutine. These titles are roughly indicative of the purpose of the sections though more information is usually produced than the title would lead one to believe.

For input the subroutine requires the direction cosines of the support linkages, the coordinates of the points of application of the links and the number of links. Appendix H

Fig. 2 Simplified Flow Chart



shows how the input must be arranged to be read in by the input-output programs of Appendix C. The output consists of the two flag arrays, KPROB and NOSOL.

The subroutine first considers the points of application of the supporting links. If these points are along a line, the question of whether it is a two- or three-dimensional problem depends on the orientation of the links. If the points are in a plane, the links must all be in that same plane for a two-dimensional problem to be involved. If the points are randomly located in space, only a three-dimensional problem can be involved. The two-dimensional problem requires three linkages for stability; the three-dimensional problem, six. If there are fewer linkages, an "unstable" error print-out is made; if more, an "indeterminate" one is made.

When it has been determined that the correct number of support linkages is involved, the orientation of the links is considered. The following restrictions apply to the three-dimensional case:

1. All the links may not be parallel.
2. They may not lie in parallel planes.
3. The axes of the links may not intersect one straight line since limited rotation would be possible about that line.
4. The axes of the links may not intersect in a point or limited rotation is possible about that point.
5. Four of the six links may not be parallel or intersect in any one point for, in these cases, a straight line can be found through the point (or infinity) which intersects the axes of the two remaining links and restriction 3. is violated.

In the two-dimensional case only restrictions 1. and 4. apply.

Where the links are all parallel to a given line, limited motion is possible in a direction perpendicular to this line.

In Fig. 3, where the pairs of links are in parallel

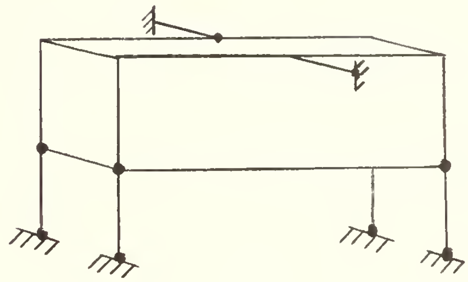


Fig. 3. Arrangement with links in parallel planes.

planes, some motion is possible in a direction perpendicular to these planes before a restoring component is developed. In Fig. 4 some rotation about the intersection

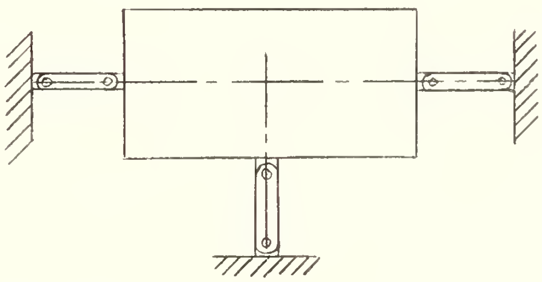


Fig. 4. Arrangement with intersecting lines of action of links.

of the lines of action of the links is necessary before a restoring couple is developed.

The following section and the appendices describe the "STADET" subroutine, its auxiliary subroutines and

the Input-Output Programs which were used to test "STADET". Fortran language was used for all the programs. The testing was done on a Control Data Corporation 1604 digital computer.

2. Method.

Subtitles indicate the sections of the flow charts (Figs. 5 through 9) and the "STADET" list (Appendix J) under discussion. Circled numbers on the flow charts are used for two purposes. The first is to facilitate the comparison with the list of Appendix J; the second is to indicate a transfer to another section of the program when the use of a line to show the path would be cumbersome.

The subroutine only operates up to the detection of the first error. The appropriate flag is then set and a return is made to the Input-Output Program.

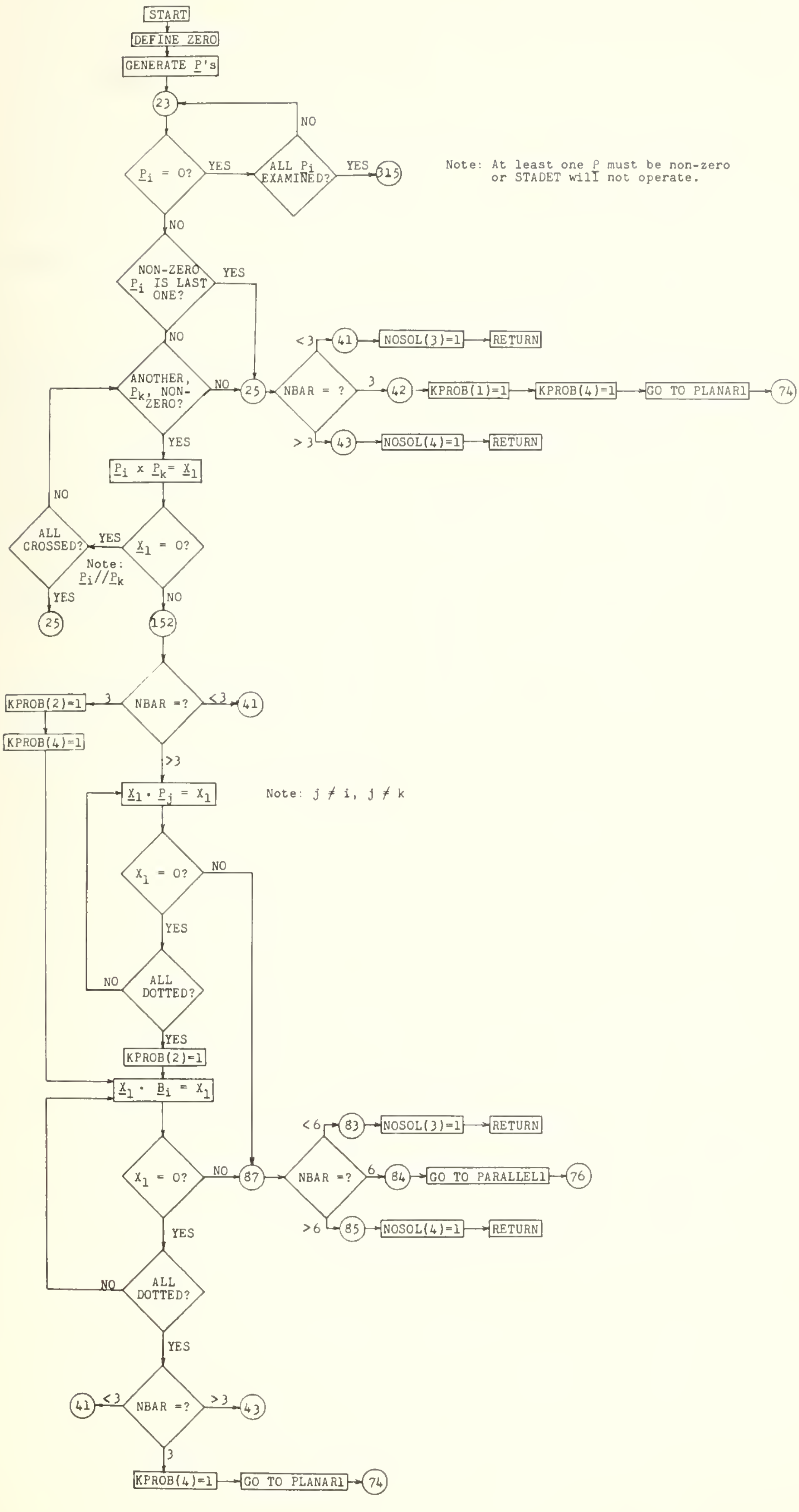
a. COPLANAR1

This section of the subroutine generates the position vectors which give the locations of the points of application of the links. It checks to see whether these points are coincident, lie on a line, are in a plane or are randomly located in space. In the case where the points of application are in a plane, this section of the program checks to see if the links are in the same plane and so determines whether a planar problem is involved.

After deciding whether a two- or three-dimensional problem is involved, this section counts the number of links to find out if the number is sufficient for stability.

The computer operates with a binary approximation to a decimal number when it is used in the floating point mode. Each calculation is approximate and round-off errors

Fig. 5 Flow Chart of COPLANAR1



can accumulate. For this reason the "zero" which must be used in the program is not the mathematical zero but an allowable tolerance. The first portion of this section examines the coordinates of the points of application and determines the value of "zero" to be used throughout "STADET". Appendix L discusses the method.

Vectors are generated between the first point of application and the other points. These vectors are examined to find the first non-zero one. If none of them is non-zero, there is only one point of support, the structure is unstable, and a return is made to the Input-Output Program.

If a non-zero vector is found, the subroutine determines whether it is the last one or not. If it is not the last, the other vectors are examined to find whether any others are non-zero. If the first non-zero vector is the last of the array of position vectors, or if all after the first are zero, the points of application of the links lie on one line.

Having determined that the points of application are on a line, the subroutine considers the number of links. A three-dimensional linkage system is unstable because the links intersect a line and only a planar problem has a unique solution. Therefore, if there are more than three links, the system is redundant. If there are less than three links, the system is unstable. If the number of links is not satisfactory, the appropriate flag is set.

Where a second non-zero vector, $\underline{P}_2$, is found, its cross product with the first, $\underline{P}_1$, is formed. The cross product vector, $\underline{X}_1$, is examined to find out if it is zero. If it is zero, $\underline{P}_1$ and $\underline{P}_2$ are parallel. Since $\underline{P}_1$ and $\underline{P}_2$ have the same initial point, this mades them coincident and indicates that the first three points of application are on a line. The subroutine then checks to see if all of the position vectors have been examined. If they have not all been examined, the search for another non-zero vector continues. If they all have been examined and all of the points of application are found to lie on one line, an investigation of the number of supporting links is made, as above.

If a cross product vector, as found above, is not zero, a check is made of the number of supporting links. If there are only three links, there are only two position vectors which (since they are not parallel) determine a plane. If there are more than three links, there are more than the two vectors that were crossed to form $\underline{X}_1$. To check whether the points of application are in a plane, it is necessary to dot these other vectors with $\underline{X}_1$. The dot product, X_1, a scalar, is compared with zero. If all the dot products are zero, the points of application lie in one plane.

With the points of application of the links in a plane a stable system is possible with either a two- or three-dimensional linkage system. If all the links lie in the

plane that contains the points of application of the links, the problem is one in two dimensions. To check for a two-dimensional problem, the vectors made up of the direction cosines of the links are dotted with $\underline{X}_1$ and the dot products are compared with zero. Since the link cosine vectors have one point in the point-of-application plane, a zero dot product indicates that the vector lies entirely in that plane. If a planar problem is what is involved, i.e., all the dot products are zero, the proper number of links is checked for, as before.

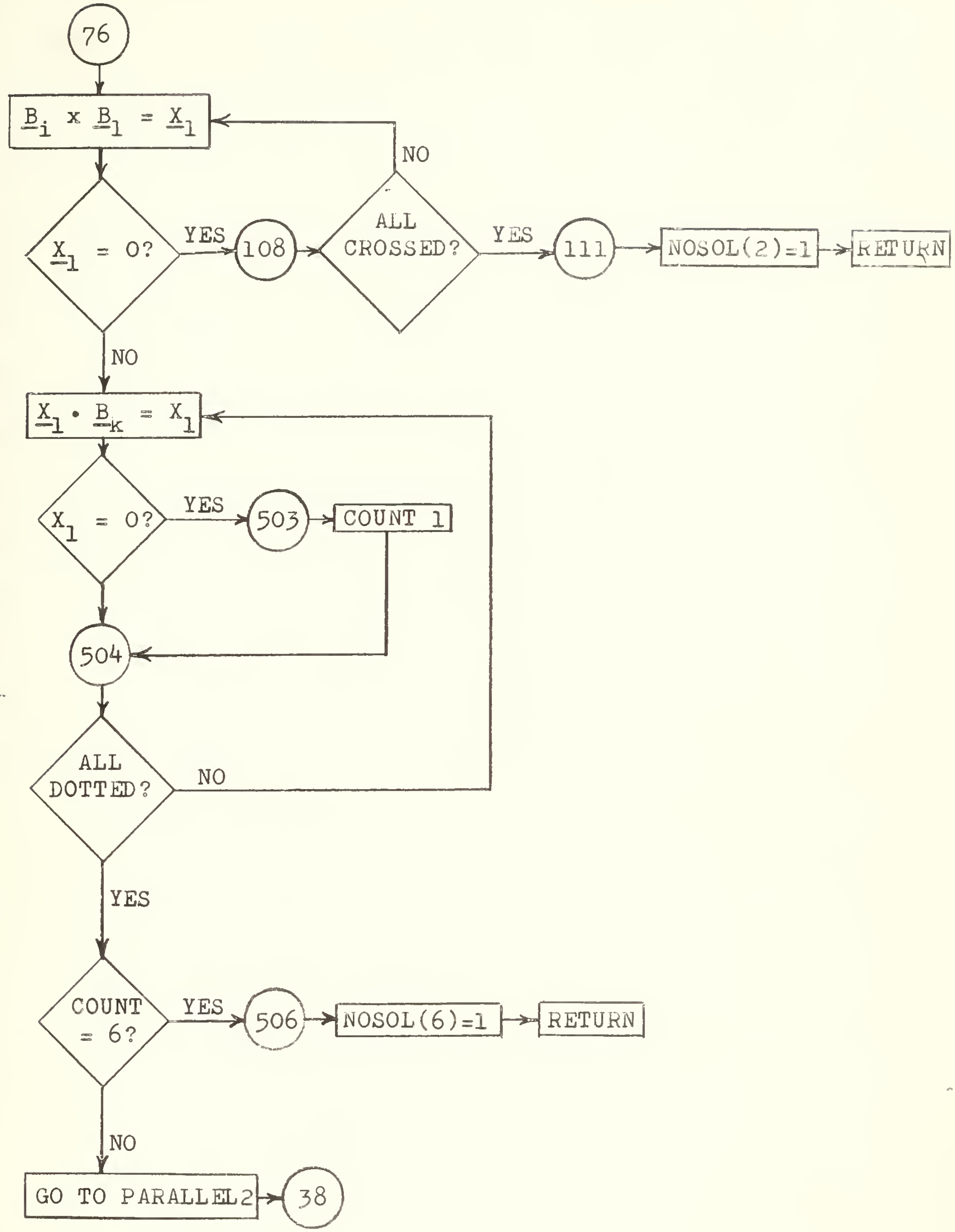
When the points of application are not in a plane or when the links are not in the point-of-application plane, a three-dimensional problem is involved and six links are required for stability. Suitable error flags are set if there are not six supports.

At this point it has been determined whether a two- or three-dimensional problem is involved and that the correct number of supporting links is involved for the type of problem. Next, instability due to incorrect orientation of the links is investigated.

b. PARALLEL1

This section checks to see if the supporting links are parallel or lie in parallel planes. The first link cosine vector is crossed with each of the succeeding ones and the resulting cross product vectors are compared with zero. If all of the cross products are zero, all of the links are parallel.

Fig. 6 Flow Chart for PARALLEL1



The second portion of this section is concerned with discovering whether a situation similar to the one shown in Fig. 3 exists or not. In the above test a non-zero cross product will result if two of the links are not parallel. (If the situation is like that of Fig. 3, the cross product vector is in the direction of motion of the rigid body.) If all of the links are perpendicular to this cross product vector, the case is indeed similar to that of Fig. 3. This section checks whether the links are perpendicular by dotting each of the link cosine vectors with the cross product vectors and comparing the resultant scalar with zero. If all of the dot products are zero, all of the link cosine vectors are in parallel planes. If all are in parallel planes, the appropriate flag is set; if not, the program goes on to the next section.

c. PARALLEL2

Even though all of the links are not parallel, the supporting linkage system is unstable, by criterion 3., if four of the six links are parallel. Therefore, a separate check is made for this possibility. The cross product test is used along with a counting system. If the linkage system passes this test, the remaining check for a three-dimensional system is done by CONCURRENT1.

d. PLANAR1

This section of the subroutine checks the three-link cases to see if the links are in a plane and if the

Fig. 7 Flow Chart for PARALLEL2

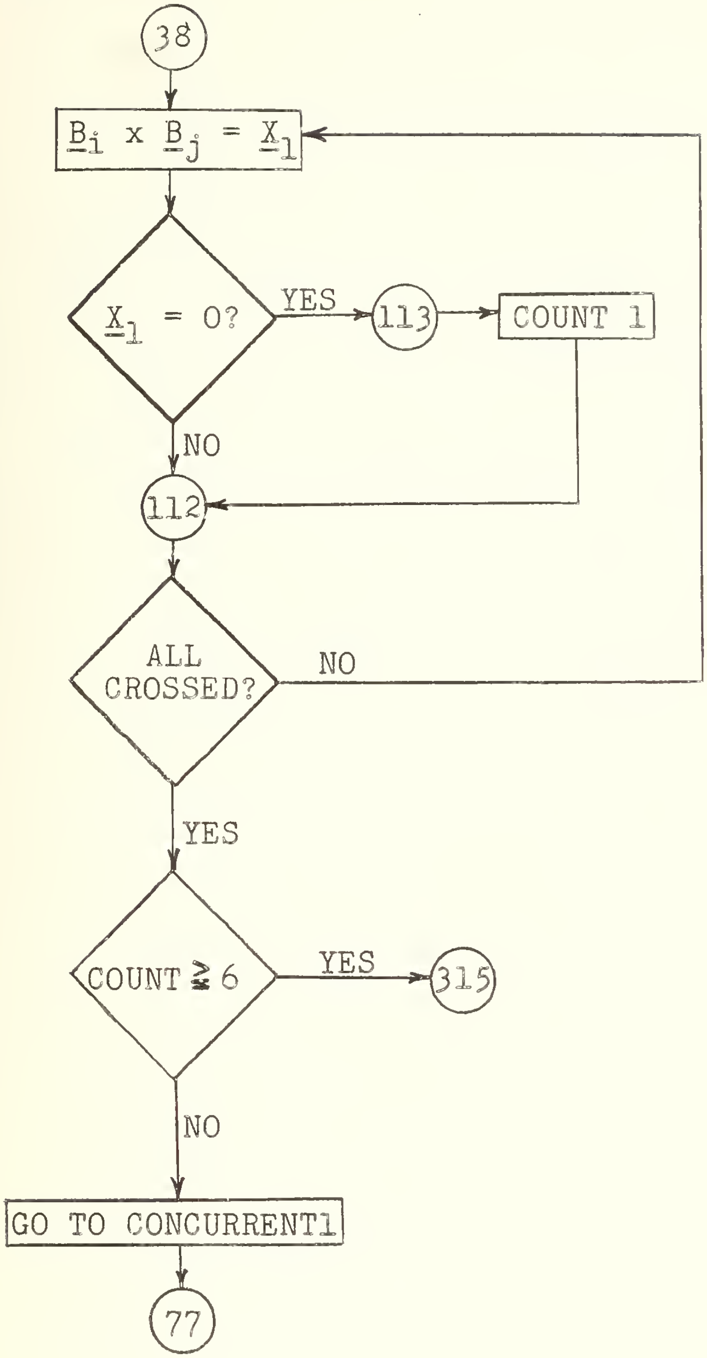
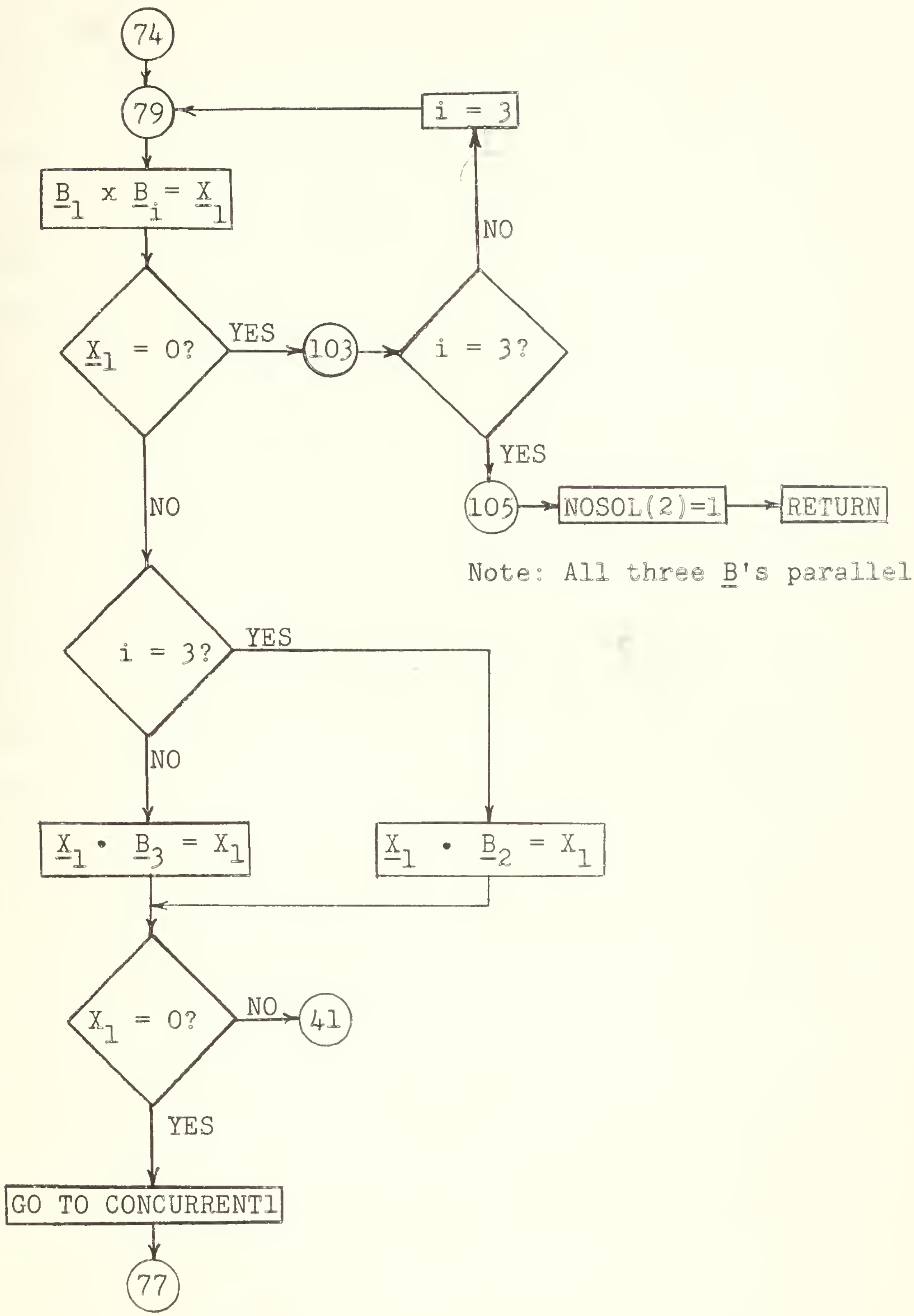


Fig. 8 Flow Chart of PLANAR1



links are parallel. In Fig. 2 there are two paths from COPLANAR1 to PLANAR1. The first of these three-link cases has the points of application on a line. In the other case the points of application are in a plane and it is known that the links are in the same plane. For this second case there is some duplication of testing, but the extra operating time is negligible and there was some convenience in programming this way.

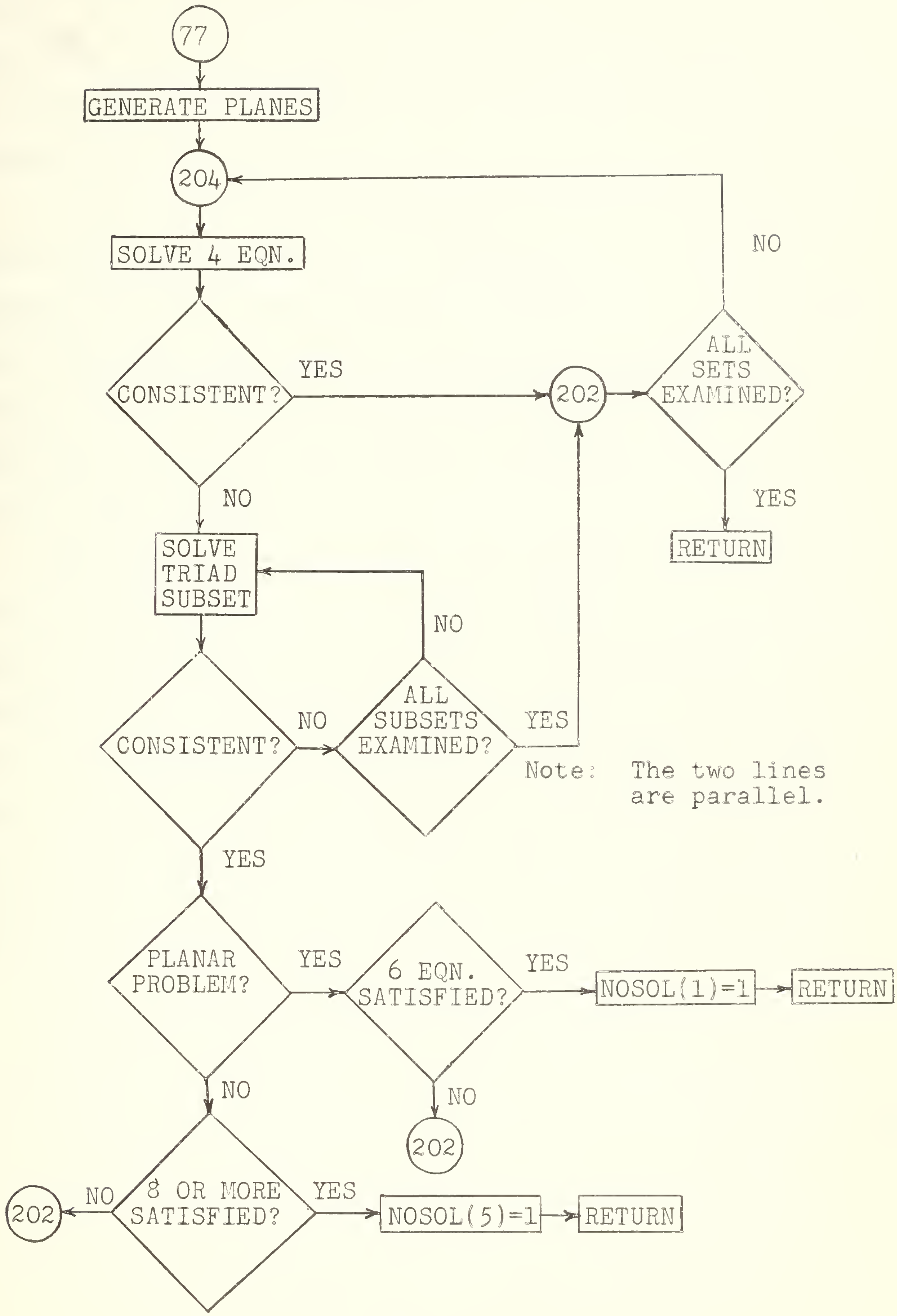
This section crosses the first link cosine vector with the second. If the cross product is zero, the first is crossed with the third link cosine vector. If this cross product is also zero, the three links are parallel and the structure is unstable. If the second cross product is not zero, it is dotted with the second link cosine vector. If the dot product is zero, all three link cosine vectors are in the same plane, a stable planar problem is possible and the program goes to CONCURRENT1. If the dot product is not zero, the three links are not coplanar and the system is unstable.

When the first cross product is not zero, it is dotted with the third link cosine vector and the dot product is compared with zero. If the dot product is not zero, the system is unstable, as above. If the dot product is zero, the program goes to CONCURRENT1.

e. CONCURRENT1

The remaining check is to find if the lines of action of the links intersect at a point. In order to

Fig. 9 Flow Chart of CONCURRENT1



be able to operate mathematically, it is necessary to convert the link cosine vectors to lines in space. The line form which was chosen was that of the intersection of two planes. (See Appendix G for discussion of subroutine PLANE.)

All of the planes are determined and the coefficients are stored in the array PLA. The equations in PLA are examined four at a time but with an overlap, i.e., equations one through four are examined first, then equations three through six; and, if a six-link problem which gives 12 equations is involved, the process continues until all 12 have been examined.

The three-link system must have at least one intersection or the links would all be parallel, a possibility which has previously been eliminated. In a six-link problem at least four of the lines of action of the links must intersect to make the linkage system unstable. Therefore, the system of examining pairs of lines whose equations are adjacent in the array PLA must find the invalidating intersection and a more complicated checking system is not required.

The coefficients of four equations are treated as though they were the coefficients of four simultaneous equations in four unknowns. They undergo the matrix singularity test of REAC1 (See Appendix F). If the determinant of the matrix is not zero, the set of equations is consistent and the first two lines do not intersect.

If the next overlapping set of four is also consistent, three lines do not intersect and the link support system is satisfactory. This would complete the checks by "STADET".

On the other hand, if the determinant is zero, the set of equations is dependent or inconsistent and the two lines intersect or are parallel. If the lines intersect, the point of intersection can be found by solving three of the equations simultaneously. If the lines are parallel, the four combinations of these four equations, taken three at a time, are all inconsistent. No account is taken of this parallelism since it is already known that less than four links are parallel. The next overlapping set of four equations is then considered.

When an intersection is found, the coordinates are substituted into the equations of all the planes. The number of equations which are satisfied is counted. In the case of a planar problem, if six are satisfied, all the lines of action intersect and the linkage system is unstable. In the three-dimensional case, if eight or more are satisfied, the linkage system is unstable. Where instability exists, the appropriate flag is set.

This completes the checks made by "STADET". The rigid body is supported by a system of links whose reactions can be determined by simple statics or the appropriate flag has been set in NOSOL.

The one remaining possible error, that of not having the load in the correct plane for a planar problem, is not

strictly of concern in deciding whether the system of links is stable and determinate. The check for this error is discussed in Appendix C, Input-Output Programs.

3. Testing of Program.

In order to devise a logical system of tests the simplified flow chart of Fig. 2 was used. This shows the questions that the various sections of the subroutine answer. In all, there are 25 distinct paths that can be followed. Only four of these paths lead to a satisfactory problem, i.e., one which can be solved by simple statics. Fig. 10 lists the types of problems which were solved. The complete print-outs of the solved problems are collected in Appendix K.

Problems 26 through 29 are the complements of the satisfactory problems of the first 25. The "bar reactions" output of TEST1A was used as the "allowable bar loads" of TEST1B. The output of TEST1B, the "allowable load torsor*" should be, and is, the "resultant torsor" of TEST1A.

In problems 11 and 17, where there are more than six links, the storage process is such that the last link is stored on top of the first. This results in a nonsense print-out of the structure data. In problem 11, for example, the first link has two direction cosines of one.

In the use of TEST1A to solve a planar problem it is necessary to consider the possibility of having an invalid problem because the load is not in the same plane as the structure. Since this possibility is caused by an

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*Torsor -- A coined word (combination of torque and tensor) used to indicate a generalized resultant which is composed of a couple and a force.

Fig. 10. List of Problems

Problem number	Unstable	Redundant	Satisfactory	Points of application on a line	Points of application coplanar	Comments
1	x					Points are concurrent
2	x			x		Less than three links (supports)
3		x		x		More than three links
4	x			x		Links are parallel
5	x			x		Links are not in one plane
6	x			x		Lines of action of links intersect
7		x			x	More than three links
8	x				x	Links are parallel
9	x				x	Lines of action of links intersect
10	x				x	Less than six links
11		x			x	More than six links
12	x				x	Links are parallel
13	x				x	Links are in parallel planes
14	x				x	Four of six links are parallel
15	x				x	Lines of action of links intersect
16	x					Less than six links
17		x				More than six links
18	x					Links are parallel
19	x					Links are in parallel planes
20	x					Four of six links are parallel
21	x					Lines of action of links intersect
22a			x	x		
22b	x			x		Resultant force not in plane of structure
23a			x		x	
23b	x				x	Resultant force not in plane of structure
23c	x				x	Resultant force not exactly in plane of structure
24			x		x	
25			x			
26			x	x		
27			x		x	Oblique plane problem
28			x		x	Stable spatial problem
29			x			Stable spatial problem

error on the part of the one setting up the problem and is not strictly a factor in the stability of the structure, testing for this possibility was made part of the Input-Output Program. The testing of a planar problem therefore requires one step beyond "STADET". The subcases of problems 22 and 23 show the results of such tests.

It was not possible to get an accurate estimate of the time required to solve a problem. It takes roughly one second to solve an unsatisfactory problem and about twice as long to solve a satisfactory one. This difference is not in the computation time but in the time required to write the output on the magnetic tape. In fact, when a series of problems is being solved, only a slight pause can be detected between the end of writing one answer and the beginning of the output for the next. To these computing and writing times must be added the time to compile the program. This is about 75 seconds for TEST1A and about 45 seconds for TEST1B.

4. Conclusion.

When this project was undertaken, it was understood that this subroutine would eventually be used as part of a much more ambitious program. The objective of providing a routine which decides whether the structure may be analyzed by the methods of simple statics has been achieved.

For the purposes of this routine a decision was made to have storage for only six equivalent links, the maximum number compatible with a solution obtainable from statics. This is the only restriction which must be removed in future, expanded use. The arrays KPROB (kind of problem) and NOSOL (no solution) were set up to be used as keys to select the proper methods of attack. If NOSOL(4) is set to one, a redundant structure is involved and more powerful methods must be used to effect a solution. KPROB indicates whether the simplification of a planar problem is valid.

While it is probable that a bona fide planar problem would be set up in one of the coordinate planes, it might be advisable to provide a subroutine to transform the coordinate system so that an oblique plane problem is transferred to a coordinate plane. It may or may not be desirable to transform the results back to the original coordinate system.

The next step in the generalized analysis of structures by digital computers could be along one of several paths. Since this subroutine is concerned with only the external

stability of the structure, a subroutine to examine the internal stability needs to be written. The simplification of considering only space frames might yield a good first approximation.

Another path would be to either use a code or have an examining type of subroutine to decide the type of problem and then jump to one of a set of subroutines designed to solve a given class of problems.

It is felt that this is one small step in removing some of the drudgery from engineering design.

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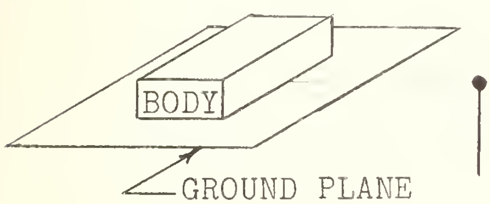
APPENDIX A

Linkage Diagrams of Common Supports

A link is a member capable of supporting only an axial force. It has spherical hinges at both ends so that it cannot transmit any moments. In the drawings below the link is shown thus:  , where the dot indicates the hinge at the point of application. The other end of the link is to be understood to have a hinge which, in turn, is attached to a frame of reference.

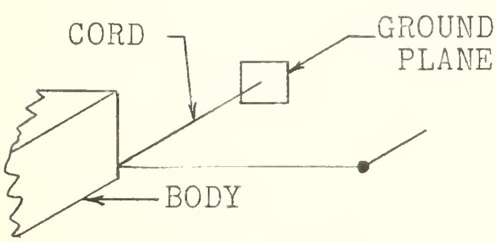
As discussed in Section 1, Introduction, some of these supports are not the equivalent of a complete link. In these cases it is the user's responsibility to either modify the support or to make sure that the reaction is in the allowed direction.

Smooth plane support



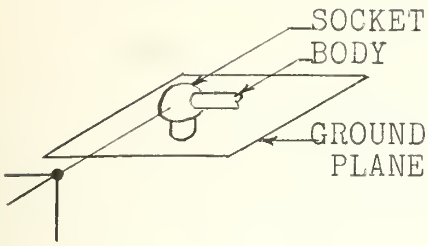
Because friction is neglected, a linkage system which only opposes vertical motion is obtained. The link shown may only have a compressive reaction.

Flexible cord



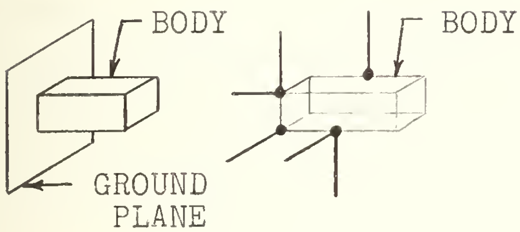
The reaction is along the cord. Only a tensile reaction is allowable.

Ball and socket



A ball and socket is the equivalent of three links acting at a point.

Built-in support



A built-in support is the equivalent of six links. The transparent body is shown supported by one of many stable and determinate arrangements of six links.

APPENDIX B

Mathematical Tests

The examination of the system of supporting links is accomplished by the application of a few simple mathematical tests to the vectors which are part of or are generated from the input data.

A. The dot or inner product of two vectors $\underline{A}$ and $\underline{B}$, neither of which is zero, is zero if and only if $\underline{A}$ and $\underline{B}$ are perpendicular.

$\underline{A} \cdot \underline{B} \equiv |\underline{A}|\ |\underline{B}| \cos \alpha$ where α = included angle

$\underline{A} \cdot \underline{B} = A_xB_x + A_yB_y + A_zB_z$ subscripts indicate x, y, z components of $\underline{A}$ and $\underline{B}$

B. The cross or outer product of two vectors $\underline{A}$ and $\underline{B}$, neither of which is zero, is zero if and only if $\underline{A}$ and $\underline{B}$ are parallel or collinear.

$\underline{A} \times \underline{B} \equiv |\underline{A}|\ |\underline{B}| \sin \alpha\ \underline{u}$ where $\underline{u}$ is a unit vector perpendicular to both $\underline{A}$ and $\underline{B}$ and with them forming a right-handed set

$$\underline{A} \times \underline{B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$\underline{i}$, $\underline{j}$ and $\underline{k}$ are unit, orthogonal base vectors in the x, y, z directions, respectively

C. The question of whether of not lines intersect is solved by examining the equations of the planes determining the lines. If a set of four equations determining a pair of lines is inconsistent or dependent, the lines are parallel or intersect. The "matrix singularity test" of REAC1 (see Appendix F) is used to find if the equations are dependent

or inconsistent.

If the lines intersect, the point of intersection can be found by solving three of the four equations simultaneously.

APPENDIX C

Input-Output Programs

These control programs provide for reading in data, operating the required subroutines and printing results. In both cases the first step is reading in the data on the equivalent linkages. Then "STADET" is called and the stability and determinateness of the support system is examined. The rest of the data for that problem is then read in regardless of the results of "STADET". The intermediate operations of the two programs will be considered separately.

TEST1A

If the structure is stable and determinate, the resultant of the forces and couples acting on the structure is calculated. This establishes the B vector in the matrix equation, $Ax = B$. The A matrix, that array which, when used to premultiply the link reaction vector, x, gives B, is found next. If there are six links, A is six by six; the six equations represented are independent and the values of the link reactions are found by subroutine REAC1.

In the case of a planar problem, "STADET" has examined the structure and found it satisfactory, but it remains to be seen whether the load is applied in the correct plane. Therefore, checks are made to see that the resultant force vector is in the plane of the structure and that the re-

sultant couple vector is perpendicular to that plane.

If there are only three links, A is six by three; only three independent equations exist among the six, and it is necessary to solve various sets of three equations until a consistent set is found (See Appendix F, REAC1). When a consistent set is found, the results are printed out as discussed below.

TEST1B

If the structure is stable and determinate, the A matrix is calculated. The allowable loads in the various links are knowns in this problem. Therefore, the x vector is multiplied by A giving the allowable load on the structure. In this case there is no question about the plane of the resultant load as in TEST1A.

Output

In both programs the intermediate calculations are suppressed if the structure is not stable and determinate. The printout then consists of the input structure, a description of the problem type and the error. If the structure is satisfactory, the input for the intermediate calculations and their results are printed. Since the case of coincident support points would be an unusual blunder, a special error printout was not used. Since "STADET" is not really operated in this case, the printouts are:

NOT A PLANAR PROBLEM and

STRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS
INTERSECT A LINE

(See problem 1.)

Both programs reset conditions so as to solve the next problem and then continue until all solutions have been made. (See Appendix H for problem format.)

```
      PROGRAM TESTIA
      DIMENSION VB(6,3),B(6,3),FORCE(100,3),CORD(100,3),COUPL(50,3),
     1KPROB(5),NOSOL(6),RFOR(3),RCOPL(3),BREAC(6),COMX(6,7),PS(6,7),
     2AR(3),BR(3),CROSS(3),VP(5,3),POINT(3,4),PLA  (12,4),PL(4),PM(4)
      COMMON VB,B,FORCE,CORD,COUPL,NFO,NCO,NBAR,KPROB,NOSOL,RFOR,RCOPL,
     1BREAC,COMX,VP
    5 FORMAT(I5)
   10 FORMAT(I3,I2)
   19 FORMAT(3F10.8,3F14.7)
   20 FORMAT(6F12.6)
   30 FORMAT(3F14.7)
   90 FORMAT(/////)
  400FORMAT(80H
     1                               ////)
  101 FORMAT(5X, 48HEXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE//)
 1110FORMAT(11X,25HDIRECTION COSINES OF BARS,21X,
     121HPOINTS OF APPLICATION)
  121 FORMAT (8X,1HL,12X,1HM,12X,1HN,13X,1HX,16X,1HY,16X,1HZ)
  129 FORMAT(3(2X,F11.7),3(3X,F14.7))
   91 FORMAT(//)
 5100FORMAT(5X, 61HSTRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS ARE C
     1ONCURRENT)
```

```
5100FORMAT(5X, 61HSTRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS ARE C
   1ONCURRENT)
5110FORMAT(5X,59HSTRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS ARE PA
   1RALLEL)
512 FORMAT(5X,50HSTRUCTURE UNSTABLE INSUFFICIENT NUMBER OF SUPPORTS)
513 FORMAT(5X,49HSTRUCTURE INDETERMINATE EXCESS NUMBER OF SUPPORTS)
5140FORMAT(5X,63HSTRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS INTERS
   1ECT A LINE)
515 FORMAT(5X,27HREACTION MATRIX IS SINGULAR)
516 FORMAT(5X,28HSUPPORT POINTS ARE ON A LINE)
517 FORMAT(5X,29HSUPPORT POINTS ARE IN A PLANE)
518 FORMAT(5X,14HPLANAR PROBLEM)
519 FORMAT(5X,43HRESULTANT LOAD IS NOT IN PLANE OF STRUCTURE)
521 FORMAT(5X,20HNOT A PLANAR PROBLEM)
522 FORMAT(5X,51HSTRUCTURE UNSTABLE  SUPPORTS ARE IN PARALLEL PLANES)
100 FORMAT(5X, 48HFIND THE RESULTANT TORSOR OF THE FOLLOWING LOADS//)
110 FORMAT(21X,6HFORCES,38X,10HAPPLIED AT)
120 FORMAT (8X,2HFX,13X,2HFY,13X,2HFZ,14X,1HX,14X,1HY,14X,1HZ)
130 FORMAT(6(3X,F12.5))
115 FORMAT(24X,7HCOUPLES)
125 FORMAT(10X,2HMX,15X,2HMY,15X,2HMZ)
135 FORMAT(3(3X,F14.7))
```

```
 51 FORMAT(5X,24HTHE RESULTANT TORSOR IS,//)
 50 FORMAT(5X,5HF = (,  F14.7,7H) I + (,F14.7,7H) J + (,F14.7,3H) K)
 55 FORMAT(5X,5HM = (,  F14.7,7H) I + (,F14.7,7H) J + (,F14.7,3H) K)
134 FORMAT(5X,21HTHE BAR REACTIONS ARE)
133 FORMAT(5X,6F14.7)
136 FORMAT(1H1)
    NP=0
    READ 5,NPROB
520 READ 40
    READ 5, NBAR
    READ 19, ((VB(I,J),J=1,3),(B(I,JJ),JJ=1,3),I=1,NBAR)
    CALL STADET
    READ 10,NFO,NCO
    IF(NFO) 590,600,590
590 CONTINUE
    READ 20, ((FORCE(I,J),J=1,3),(CORD(I,JJ),JJ=1,3),I=1,NFO)
600 CONTINUE
    IF(NCO) 610,620,610
610 CONTINUE
    READ 30, ((COUPL(I,J),J=1,3),I=1,NCO)
620 CONTINUE
    DO 1 I = 1,6
    ND = NOSOL(I)+1
    GO TO (1,2),ND
```

```
    1 CONTINUE
      CALL RES1
      DO 200 I = 1,3
      DO 200 J = 1,NBAR
  200 COMX(I,J) = VB(J,I)
      DO  201 I  = 1,NBAR
      COMX(4,I) = -VB(I,2)*B(I,3)+VB(I,3)*B(I,2)
      COMX(5,I) = -VB(I,3)*B(I,1)+VB(I,1)*B(I,3)
  201 COMX(6,I) = -VB(I,1)*B(I,2)+VB(I,2)*B(I,1)
      SUM = 0.
      DO 202 I = 1,3
      SUM = SUM + RFOR(I)
  202 COMX(I,NBAR+1) = RFOR(I)
      SUM2 = 0.
      DO 203 I = 4,6
      SUM2 = SUM2 + RCOPL(I-3)
  203 COMX(I,NBAR+1) = RCOPL(I-3)
      IF (SUM-SUM2) 613,613,614
  614 ZR = (SUM/3.)*1.E-10
      GO TO 615
  613 ZR = (SUM2/3.)*1.E-10
  615 CONTINUE
      IF (KPROB(4)) 302,301,302
  301 NUNC = 6
```

```
      CALL REAC1(COMX,BREAC,NUNC,ZR,KPROB(3))
      GO TO 2
  302 CONTINUE
      EP = .00000001
      IF (NFO) 609,604,609
  609 DO 601 I = 1,3
      AR(I) = VB(1,I)
  601 BR(I) = VB(2,I)
      CALL CROST (AR,BR,CROSS)
      SUM = 0.
      DO 602 I = 1,3
  602 SUM = SUM + RFOR(I)*RFOR(I)
      DEN = SQRTF(SUM)
      DO 603 I = 1,3
  603 AR(I) = RFOR(I)/DEN
      C = 0.
      DO 606 I = 1,3
  606 C = C + CROSS(I)*AR(I)
      IF (ABSF(C)-EP) 604,604,605
  605 KPROB(5) = 1
      GO TO 2
  604 CONTINUE
      IF (NCO) 612,611,612
  612 SUM = 0.
```

```
      DO 607 I = 1,3
  607 SUM = SUM + RCOPL(I)*RCOPL(I)
      DEN = SQRTF(SUM)
      DO 608 I = 1,3
  608 AR(I) = RCOPL(I)/DEN
      CALL CROST (AR,CROSS,BR)
      IF((ABSF(BR(1))+ABSF(BR(2))+ABSF(BR(3)))-EP) 611,611,605
  611 CONTINUE
      I1=1
      J1=2
      K1=3
      NUNC =3
  303 DO 304 I = 1,4
      PS(1,I) = COMX(I1,I)
      PS(2,I) = COMX(J1,I)
  304 PS(3,I) = COMX(K1,I)
      CALL REAC1(PS,BREAC,NUNC,ZR,KM)
      IF (KM) 306,2,306
  306 IF (K1-6) 308,309,309
  308 K1 = K1+1
      GO TO 303
  309 IF (J1-5) 310,311,311
  310 J1=J1+1
      K1 = J1+1
```

```
      GO TO 303
  311 IF (I1-4) 312,313,313
  312 I1 = I1+1
      J1 =I1+1
      K1 = J1+1
      GO TO 303
  313 KPROB(3) = 1
    2 CONTINUE
      PRINT 90
      PRINT 40
      PRINT 101
      PRINT 111
      PRINT 121
      PRINT 129,((VB(I,J),J=1,3),(B(I,JJ),JJ=1,3),I=1,NBAR)
      PRINT 91
      IF (KPROB(1)) 157,158,157
  157 PRINT 516
  158 IF (KPROB(2)) 159,165,159
  159 PRINT 517
  165 IF(KPROB(3)) 161,162,161
  161 PRINT 515
  162 IF (KPROB(4)) 163,168,163
  163 PRINT 518
      GO TO 164
```

```
168 PRINT 521
164 IF(KPROB(5)) 167,166,167
167 PRINT 91
    PRINT 519
    GO TO 320
166 CONTINUE
    PRINT 91
    I1=0
    DO 102 I = 1,6
    I1 = I1+1
    IF (NOSOL(I)) 112,102,112
102 CONTINUE
    GO TO 321
112 GO TO (500,501,502,503,504,505),I1
500 PRINT 510
    PRINT 91
    GO TO 320
501 PRINT 511
    PRINT 91
    GO TO 320
502 PRINT 512
    PRINT 91
    GO TO 320
503 PRINT 513
```

```
      PRINT 91
      GO TO 320
  504 PRINT 514
      PRINT 91
      GO TO 320
  505 PRINT 522
      PRINT 91
      GO TO 320
  321 CONTINUE
      PRINT 100
      IF (NFO) 316,317,316
  316 CONTINUE
      PRINT 110
      PRINT 120
      PRINT 130,((FORCE(I,J),J=1,3),(CORD(I,JJ),JJ=1,3), I=1,NFO)
      PRINT 91
      DO 560 I=1,NFO
      DO 560 J=1,3
      FORCE(I,J)=0.
  560 CORD(I,J)=0.
  317 CONTINUE
      IF (NCO) 314,315,314
  314 CONTINUE
      PRINT 115
```

```
      PRINT 125
      PRINT 135,((COUPL(I,JJJ),JJJ=1,3),I=1,NCO)
      PRINT 91
      DO 570 I=1,NCO
      DO 570 J=1,3
  570 COUPL(I,J)=0.
  315 CONTINUE
      PRINT 51
      PRINT 50,(RFOR(J),J=1,3)
      PRINT 55,(RCOPL(J),J=1,3)
      PRINT 91
      PRINT 134
      PRINT 133,(BREAC(I),I=1,6)
  320 CONTINUE
      PRINT 90
      DO 16 I=1,5
   16 KPROB(I) = 0
      DO 17 I = 1,6
   17 NOSOL(I) = 0
      DO 571 I = 1,NBAR
      DO 571 J = 1,3
      B(I,J) = 0.
  571 VB(I,J) = 0.
      DO 572 I = 1,6
```

```
572 BREAC(I) = 0.
    NP=NP+1
    PRINT 136
    IF(NP-NPROB) 520,580,580
580 CONTINUE
    STOP
    END
```

```
      PROGRAM TESTIB
      DIMENSION VB(6,3),B(6,3),KPROB(5),NOSOL(6),RFOR(3),RCOPL(3),
     1BREAC(6),COMX(6,7),AR(3),BR(3),CROSS(3),VP(5,3),POINT(3,4),
     2PLA(12,4),PL(4),PM(4),BFORCE(6),PS(6,7)
      COMMON VB,B,NBAR,KPROB,NOSOL,COMX,VP,BFORCE
    5 FORMAT(I5)
   19 FORMAT(3F10.8,3F14.7)
   20 FORMAT(6F12.6)
   90 FORMAT(/////)
  400 FORMAT(80H
     1                             ///)
  101 FORMAT(5X, 48HEXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE//)
  1110FORMAT(11X,25HDIRECTION COSINES OF BARS,21X,
     121HPOINTS OF APPLICATION)
  121 FORMAT (8X,1HL,12X,1HM,12X,1HN,13X,1HX,16X,1HY,16X,1HZ)
  129 FORMAT(3(2X,F11.7),3(3X,F14.7))
  130 FORMAT(5X,19HALLOWABLE BAR LOADS//)
  131 FORMAT(6F14.7)
   91 FORMAT(//)
  5100FORMAT(5X, 61HSTRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS ARE C
     1ONCURRENT)
  5110FORMAT(5X,59HSTRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS ARE PA
     1RALLEL)
  512 FORMAT(5X,50HSTRUCTURE UNSTABLE INSUFFICIENT NUMBER OF SUPPORTS)
```

```
513 FORMAT(5X,49HSTRUCTURE INDETERMINATE EXCESS NUMBER OF SUPPORTS)
5140FORMAT(5X,63HSTRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS INTERS
   1ECT A LINE)
515 FORMAT(5X,27HREACTION MATRIX IS SINGULAR)
516 FORMAT(5X,28HSUPPORT POINTS ARE ON A LINE)
517 FORMAT(5X,29HSUPPORT POINTS ARE IN A PLANE)
518 FORMAT(5X,14HPLANAR PROBLEM)
521 FORMAT(5X,20HNOT A PLANAR PROBLEM)
522 FORMAT(5X,51HSTRUCTURE UNSTABLE  SUPPORTS ARE IN PARALLEL PLANES)
 61 FORMAT(5X,28HTHE ALLOWABLE LOAD TORSOR IS)
 50 FORMAT(5X,5HF = (,  F14.7,7H) I + (,F14.7,7H) J + (,F14.7,3H) K)
 55 FORMAT(5X,5HM = (,  F14.7,7H) I + (,F14.7,7H) J + (,F14.7,3H) K)
136 FORMAT(1H1)
    NP=0
    READ 5,NPROB
520 READ 40
    READ 5, NBAR
    READ 19, ((VB(I,J),J=1,3),(B(I,JJ),JJ=1,3),I=1,NBAR)
    CALL STADET
    READ 20, (BFORCE(I),I=1,6)
    DO 1 I = 1,6
    ND = NOSOL(I)+1
    GO TO (1,2),ND
  1 CONTINUE
```

```
      DO 200 I = 1,3
      DO 200 J = 1,NBAR
  200 COMX(I,J) = VB(J,I)
      DO  201 I  = 1,NBAR
      COMX(4,I) = -VB(I,2)*B(I,3)+VB(I,3)*B(I,2)
      COMX(5,I) = -VB(I,3)*B(I,1)+VB(I,1)*B(I,3)
  201 COMX(6,I) = -VB(I,1)*B(I,2)+VB(I,2)*B(I,1)
      DO 602 I = 1,3
      RFOR(I) = 0.
  602 RCOPL(I) = 0.
      DO 601 I = 1,3
      DO 601 J = 1,6
      RFOR(I) = RFOR(I)+BFORCE(J)*COMX(I,J)
  601 RCOPL(I) = RCOPL(I)+BFORCE(J)*COMX(I+3,J)
    2 CONTINUE
      PRINT 90
      PRINT 40
      PRINT 101
      PRINT 111
      PRINT 121
      PRINT 129,((VB(I,J),J=1,3),(B(I,JJ),JJ=1,3),I=1,NBAR)
      PRINT 91
      IF (KPROB(1)) 157,158,157
  157 PRINT 516
```

```
158 IF (KPROB(2)) 159,165,159
159 PRINT 517
165 IF(KPROB(3)) 161,162,161
161 PRINT 515
162 IF (KPROB(4)) 163,164,163
163 PRINT 518
    GO TO 322
164 PRINT 521
322 CONTINUE
    PRINT 91
    I1=0
    DO 100 I = 1,6
    I1 = I1+1
    IF (NOSOL(I)) 110,100,110
100 CONTINUE
    GO TO 321
110 GO TO (500,501,502,503,504,505),I1
500 PRINT 510
    GO TO 320
501 PRINT 511
    GO TO 320
502 PRINT 512
    GO TO 320
503 PRINT 513
```

```
      GO TO 320
  504 PRINT 514
      GO TO 320
  505 PRINT 522
      GO TO 320
  321 CONTINUE
      PRINT 91
      PRINT 130
      PRINT 131,(BFORCE(I),I = 1,6)
      PRINT 91
      PRINT 61
      PRINT 50,(RFOR(J),J=1,3)
      PRINT 55,(RCOPL(J),J=1,3)
  320 CONTINUE
      PRINT 91
      DO 16 I=1,5
   16 KPROB(I) =0
      DO 17 I = 1,6
   17 NOSOL(I) = 0
      DO 571 I = 1,NBAR
      DO 571 J = 1,3
      B(I,J) = 0.
  571 VB(I,J) = 0.
      NP=NP+1
```

```
      PRINT 136
      IF(NP-NPROB) 520,580,580
  580 CONTINUE
      STOP
      END
```

APPENDIX D

Subroutine CROST

The subroutine evaluates the cross product of two vectors expressed in rectangular coordinates. A Fortran expression is written for each term of the expansion of CROSS.

$$\text{CROSS} \equiv \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - A_z B_y)\underline{i} + (A_z B_x - A_x B_z)\underline{j} + (A_x B_y - A_y B_x)\underline{k}$$

$$= \text{CROSS}(1)\underline{i} + \text{CROSS}(2)\underline{j} + \text{CROSS}(3)\underline{k}$$

```
SUBROUTINE CROST (AR,BR,CROSS)
DIMENSION AR(3),BR(3),CROSS(3)
CROSS(1) = AR(2)*BR(3)-AR(3)*BR(2)
CROSS(2) = AR(3)*BR(1)-AR(1)*BR(3)
CROSS(3) = AR(1)*BR(2)-AR(2)*BR(1)
RETURN
END
```

APPENDIX E

Subroutine RES1

The subroutine finds the resultant of a system of forces and/or couples by first adding up the rectangular components of either and then calculating the effect, if any, of the forces on the resultant couple.

$F_x = \sum_i F_{xi}$ etc.	F_{xi} is the x component of $\underline{F}_i$
$M_x = \sum_i F_{xi}$ etc.	M_{xi} is the x component of $\underline{M}_i$
$\underline{r} = r_x\underline{i} + r_y\underline{j} + r_z\underline{k}$	$\underline{r}$ is the radius vector to the point of application of the force
	r_x, r_y and r_z are the rectangular components of $\underline{r}$
$\underline{F} = F_x\underline{i} + F_y\underline{j} + F_z\underline{k}$	$\underline{F}$ is the vector force
	F_x, F_y and F_z are the rectangular components of $\underline{F}$
$\underline{M} \equiv \underline{r} \times \underline{F}$	$\underline{M}$ is the vector moment
$= M_x\underline{i} + M_y\underline{j} + M_z\underline{k}$	M_x, M_y and M_z are the rectangular components of $\underline{M}$

$$= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

```
      SUBROUTINE RES1
      DIMENSION VB(6,3),B(6,3),FORCE(100,3),CORD(100,3),COUPL(50,3),
     1KPROB(5),NOSOL(6),RFOR(3),RCOPL(3),BREAC(6),COMX(6,7),PS(6,7),
     2AR(3),BR(3),CROSS(3),VP(5,3),POINT(3,4),PLA  (12,4),PL(4),PM(4)
      COMMON VB,B,FORCE,CORD,COUPL,NFO,NCO,NBAR,KPROB,NOSOL,RFOR,RCOPL,
     1BREAC,COMX,VP
      DO 5 I=1,3
      RCOPL(I)= 0.
    5 RFOR(I)=0.
      IF (NFO) 40,50,40
   40 CONTINUE
      DO 10 I=1,3
      DO 10 J=1,NFO
   10 RFOR(I) = RFOR(I)+ FORCE(J,I)
   50 CONTINUE
      IF(NCO) 60,70,60
   60 CONTINUE
      DO 20 J=1,3
      DO 20 I=1,NCO
   20 RCOPL(J)=RCOPL(J)+COUPL(I,J)
```

```
70 CONTINUE
   IF(NFO) 80,90,80
80 CONTINUE
   DO 30 I=1,NFO
   RCOPL(1)=RCOPL(1)-FORCE(I,2)*CORD(I,3)+FORCE(I,3)*CORD(I,2)
   RCOPL(2)=RCOPL(2)-FORCE(I,3)*CORD(I,1)+FORCE(I,1)*CORD(I,3)
30 RCOPL(3)=RCOPL(3)-FORCE(I,1)*CORD(I,2)+FORCE(I,2)*CORD(I,1)
90 CONTINUE
   RETURN
   END
```

APPENDIX F

Subroutine REAC1

This subroutine is a modification of the GAUSS2 subroutine used by C. B. Bailey[2] in his program for solving simultaneous linear equations. Whereas Bailey was solving the matrix equation, $Ax = B$, with a maximum of 50 vectors B for each coefficient matrix A, nowhere in this program is more than one B vector involved with a given A.

The routine locates the largest first column coefficient and, if necessary, exchanges the row containing this coefficient with the first so the largest element is A_{11}. Multiples of row one are subtracted from the other rows to make the column one coefficients below the first zero. Next the column two coefficients below the first are examined to find the largest and rows are exchanged, if necessary, as before. This Gaussian elimination process is continued until all the elements below the main diagonal are zero.

At each reduction step, the value of the diagonal element is compared with zero. A very small diagonal element indicates ill-conditioned equations and causes an error output. This is the "matrix singularity" test used in CONCURRENT1. If the matrix is non-singular, a back- solving process is used to find the unknowns.

---

[2] C. B. Bailey, F2 UTEX LINEQN, CO OP Manual, Dec., 1960.

```
      SUBROUTINE REAC1(AA,X,N,FP,KER)
      DIMENSION AA(6,7),X(6)
      KER = 0
      DO 4 I = 1,N
    4 X(I) = 0.
      NPM = N + 1
      DO 34 L = 1,N
      KP = C
      Z = 0.0
      DO 12 K = L,N
      IF(Z-ABSF(AA(K,L))) 11,12,12
   11 Z = ABSF(AA(K,L))
      KP = K
   12 CONTINUE
      IF(L-KP) 13,20,20
   13 DO 14 J = L,NPM
      Z = AA(L,J)
      AA(L,J) = AA(KP,J)
   14 AA(KP,J) = Z
   20 IF(ABSF(AA(L,L))-FP) 50,50,30
   30 IF (L-N) 31,40,40
   31 LP1 = L+1
      DO 34 K = LP1,N
      IF (AA(K,L)) 32,34,32
```

```
32 RATIO = AA(K,L)/AA(L,L)
   DO 33 J = LP1,NPM
33 AA(K,J) = AA(K,J) - RATIO*AA(L,J)
34 CONTINUE
40 DO 43 I = 1,N
   II = N + 1 - I
   S = 0.0
   IF (II-N) 41,43,43
41 IIP1 = II + 1
   DO 42 K = IIP1,N
42 S = S + AA(II,K)*X(K)
43 X(II)=(AA(II,NPM)-S)/AA(II,II)
   RETURN
50 KER = 1
   RETURN
   END
```

APPENDIX G

Subroutine PLANE

Given a vector with direction cosines l, m and n and one point, the problem is to find a pair of planes whose intersection is parallel to the vector and passes through the point.

First, the subroutine checks to see if the vector is "nearly" parallel to a coordinate axis. "Nearly" is defined as having a direction cosine greater than 0.8, i.e., lying within about 37° of the coordinate directions. If the vector is "nearly" parallel to one axis, it is crossed with vectors along the perpendicular axes.

If the vector is not "nearly" parallel, a check is made to see whether it lies within about 6° of a coordinate plane, i.e., the direction cosine is less than 0.1. If it is close to a coordinate plane, one of the crossing vectors is chosen perpendicular to that plane.

If neither of the above tests is met, the given vector is crossed with vectors in the x and z directions.

There are now two cross product vectors of the form:

$$\underline{X}_1 = A\underline{i} + B\underline{j} + C\underline{k}$$

Since these vectors are perpendicular to the planes containing the vectors which formed the cross products, the dot product of $\underline{X}_1$ and an arbitrary vector in one of the planes is zero. An arbitrary vector is of the form:

$$(x-x_o)\underline{i} + (y-y_o)\underline{j} + (z-z_o)\underline{k}$$

where (x_o, y_o, z_o) is the known point. The dot product has the form:

$$A(x-x_o) + B(y-y_o) + C(z-z_o) = 0$$

This may be rearranged to:

$$Ax + By + Cz = D$$

where: $D = Ax_o + By_o + Cz_o$

The coefficients A, B, C, and D are stored in the array PLA.

```
      SUBROUTINE PLANE(CC,DD,PL,PM)
      DIMENSION CC(3),DD(3),PL(4),PM(4),AA(3),BB(3)
      DO 7 K = 1,3
      AA(K) = 0.
      PL(K) = 0.
      PM(K) = 0.
    7 BB(K) = 0.
      PL(4) = 0.
      PM(4) = 0.
      DO 1 K = 1,3
      IF(ABSF(CC(K))-.8) 1,1,3
    1 CONTINUE
      DO 11 K = 1,3
      IF(ABSF(CC(K))-.1) 9,9,11
   11 CONTINUE
    5 AA(1) = 1.
      BB(3) = 1.
      GO TO 4
    6 AA(1) = 1.
      BB(2) = 1.
      GO TO 4
    2 AA(2) = 1.
      BB(3) = 1.
```

```
    4 CONTINUE
      CALL CROST(CC,AA,PL)
      CALL CROST(CC,BB,PM)
    8 DO 10 K = 1,3
      PL(4) = PL(4) + PL(K)*DD(K)
   10 PM(4) = PM(4) +PM(K)*DD(K)
      RETURN
    3 GO TO (2,5,6),K
    9 GO TO (6,2,5),K
      END
```

APPENDIX H

Problem Format

TEST1A

Fortran Format	Symbol(s)	Input data
I5	NPROB	Total number of problems
Hollerith		Problem title - up to 80 characters
I5	NBAR	Number of links
3F10.8,3F14.7	VB,B	Direction cosines, coordinates of links
I3,I2	NFO,NCO	Number of forces, couples
6F12.6	FORCE,CORD	Force components, points of application
3F14.7	COUPL	Couple components

Repeats from the Hollerith title for each problem.

TEST1B

Fortran Format	Symbol(s)	Input Data
I5	NPROB	Total number of problems
Hollerith		Problem title - up to 80 characters
I5	NBAR	Number of links
3F10.8,3F14.7	VB,B	Direction cosines, coordinates of links
6F12.6	BFORCE	Allowable link loads

Repeats from the Hollerith title for each problem

APPENDIX I

STADET Flags

KPROB = Kind of Problem

1 Support points are on a line

2 Support points are in a plane

3 Reaction matrix is singular

4 Planar problem (if 4 is 1)

Not a planar problem (if 4 is 0)

5 Resultant load is not in plane of structure

NOSOL = No solution because:

1 Structure unstable - lines of action of supports are concurrent

2 Structure unstable - lines of action of supports are parallel

3 Structure unstable - insufficient number of supports

4 Structure indeterminate - excess number of supports

5 Structure unstable - lines of action of supports intersect a line

6 Structure unstable - supports are in parallel planes

APPENDIX J

STADET List

```
      SUBROUTINE STADET
      DIMENSION VB(6,3),B(6,3),FORCE(100,3),CORD(100,3),COUPL(50,3),
     1KPROB(5),NOSOL(6),RFOR(3),RCOPL(3),BREAC(6),COMX(6,7),PS(6,7),
     2AR(3),BR(3),CROSS(3),VP(5,3),POINT(3,4),PLA  (12,4),PL(4),PM(4)
      COMMON VB,B,FORCE,CORD,COUPL,NFO,NCO,NBAR,KPROB,NOSOL,RFOR,RCOPL,
     1BREAC,COMX,VP
C     COPLANAR1
      SUM = 0.
      DO 401 I = 1,3
      DO 401 J = 1,3
  401 SUM = SUM + B(I,J)
      EP = (SUM/9.)*1.E-6
      NBA = NBAR -1
      DO 11 I = 1,NBA
      DO 11 J=1,3
   11 VP(I,J)=B((I+1),J)-B(1,J)
      I = 1
      J = 0
      J1 = 0
   23 IF (ABSF(VP(I,1))+ABSF(VP(I,2))+ABSF(VP(I,3))-EP) 22,22,21
   22 I=I+1
      IF (I-NBAR  ) 23,315,23
   21 IF (I-NBAR+1) 24,25,25
   24 K = I + 1
```

```
 26 IF (ABSF(VP(K,1))+ABSF(VP(K,2))+ABSF(VP(K,3))-EP) 32,32,31
 32 IF(K+1-NBAR) 33,25,25
 33 K=K+1
    GO TO 26
 25 IF (NBAR-3) 41,42,43
 41 NOSOL(3)=1
    RETURN
 42 KPROB(1)=1
    KPROB(4) = 1
    GO TO 74
 43 NOSOL(4)=1
    RETURN
 31 DO 5 L = 1,3
    AR(L) = VP(I,L)
  5 BR(L) = VP(K,L)
    CALL CROST (AR,BR,CROSS)
    IF (ABSF(CROSS(1))+ABSF(CROSS(2))+ABSF(CROSS(3))-EP) 51,51,152
 51 IF (K+1-NBAR) 61,25,25
 61 K=K+1
    GO TO 26
152 IF(NBAR-3)41,156,52
156 KPROB(2) = 1
    KPROB(4) = 1
    GO TO 73
```

```
52 J = J+1
   IF (J-I) 53,52,53
53 IF (J-K)54,52,54
54 C =CRCSS(1)*VP(J,1)+CROSS(2)*VP(J,2)+CROSS(3)*VP(J,3)
   J1 = J1+1
   IF (ABSF(C)-EP) 62,62,81
62 IF (J1+3-NBAR)  52,72,72
72 KPROB(2)=1
   I=1
73 C=CROSS(1)*VB(I,1)+CROSS(2)*VB(I,2)+CROSS(3)*VB(I,3)
   IF (ABSF(C)-EP) 82,82,81
81 IF (NBAR-6)  83,76,85
85 NOSOL(4)=1
   RETURN
83 NOSOL(3)=1
   RETURN
82 IF(I-NBAR) 86,87,87
86 I=I+1
   GO TO 73
87 IF(NBAR-3) 41,88,43
88 KPROB(4) = 1
```

```
C       PLANAR1
    74 I=2
    79 DO 6 L = 1,3
       AR(L) = VB(1,L)
     6 BR(L) = VB(I,L)
       CALL CROST (AR,BR,CROSS)
   101 IF (ABSF(CROSS(1))+ABSF(CROSS(2))+ABSF(CROSS(3))-EP) 103,103,102
   103 IF(I-3) 104,105,105
   104 I=3
       GO TO 79
   105 NOSOL(2) = 1
       RETURN
   102 IF (I-3) 106,176,176
   106 C=CROSS(1)*VB(3,1)+CROSS(2)*VB(3,2)+CROSS(3)*VB(3,3)
       IF (ABSF(C)-EP) 77,77,41
   176 C=CROSS(1)*VB(2,1)+CROSS(2)*VB(2,2)+CROSS(3)*VB(2,3)
       IF (ABSF(C)-EP) 77,77,41
```

```
C       PARALLEL1
   76 I=1
  109 I=I+1
      K = 0
      M = 0
      DO 7 L = 1,3
      AR(L) = VB(1,L)
    7 BR(L) = VB(I,L)
      CALL CROST (AR,BR,CROSS)
      IF(ABSF(CROSS(1))+ABSF(CROSS(2))+ABSF(CROSS(3))-EP) 108,108,501
  108 IF(I-NBAR) 109,111,111
  111 NOSOL(2)=1
      RETURN
  501 C = 0.
      K = K+1
      DO 502 L = 1,3
      C = C + CROSS(L)*VB(K,L)
  502 CONTINUE
      IF (ABSF(C)-EP) 503,503,504
```

```
503 M = M+1
504 CONTINUE
    IF (K-NBAR) 501,505,505
505 IF (M-6) 38,506,506
506 NOSOL(6) = 1
    RETURN
```

```
C       PARALLEL2
   38 I=1
      J=2
      I1=0
  116 DO 8 L = 1,3
      AR(L) = VB(I,L)
    8 BR(L) = VB(J,L)
      CALL CROST (AR,BR,CROSS)
      IF(ABSF(CROSS(1))+ABSF(CROSS(2))+ABSF(CROSS(3))-EP) 113,113,112
  113 I1=I1+1
  112 IF(J-6) 114,115,115
  114 J=J+1
      GO TO 116
  115 IF(I-5) 118,117,117
  118 I=I+1
      J = I+1
      GO TO 116
  117 IF(I1-6) 77,315,315
```

```
C       CONCURRENT1
   77 LL = 0
  402 LL = LL + 1
      DO 1 N = 1,3
      AR(N) = VB(LL,N)
    1 BR(N) =  B(LL,N)
      CALL PLANE (AR,BR,PL,PM)
      DO 316 I = 1,4
      KO = 2*LL
      KI = KO-1
      PLA (KI,I) = PL(I)
  316 PLA(KO,I) = PM(I)
      IF (LL-NBAR) 402,2,2
    2 CONTINUE
      K1 = 0
      K2 = 4
  204 KL = 4
      DO 3 K = 1,4
      DO 3 L = 1,4
    3 COMX(K,L) = PLA(K+K1,L)
      CALL REAC1(COMX,BREAC,KL,EP,KM)
      IF (KM) 201,202,201
  202 IF(K2-2*NBAR) 203,205,205
  203 K1 = K1+2
```

```
      K2 = K2+2
      GO TO 204
  205 RETURN
  201 CONTINUE
      KL = 3
      I1 = 1
      I2 = 2
      I3 = 3
  208 DO 206 L = 1,4
      PS(1,L) = COMX(I1,L)
      PS(2,L) = COMX(I2,L)
      PS(3,L) = COMX(I3,L)
  206 CONTINUE
      CALL REAC1(PS,BREAC,KL,EP,KM)
      IF(KM) 207,4,207
  207 CONTINUE
      I3 = I3+1
      IF (I3-4)  208,208,209
  209 I2 = I2+1
      I3 = I2+1
      IF (I3-4)  208,208,210
  210 I1 = I1+1
      I2 =I1+1
      I3 = I2+1
```

```
      IF (I3-4)  208,208,202
    4 CONTINUE
      I5 = 0
      I6 = 0
  213 I5 = I5+1
      DD = 0.
      DO 314 K = 1,3
  314 DD = DD + BREAC(K)*PLA(I5,K)
      RID = PLA(I5,4) - DD
      IF (ABSF(RID)-FP) 312,312,311
  312 I6 = I6+1
  311 IF(I5-2*NBAR) 213,317,317
  317 IF(NBAR-3) 318,318,313
  318 IF (I6-6) 202,321,321
  321 NOSOL(1) = 1
      RETURN
  313 IF (I6-8) 202,315,315
  315 NOSOL(5) = 1
      RETURN
      END
```

APPENDIX K

Problem Types

In this appendix "bars" and "supports" are used as synonyms for "links". See Section 3, Testing of Program, for a discussion of the problems.

1. UNSTABLE STRUCTURE - COINCIDENT POINTS OF SUPPORT

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
-.7388637	.6562276	.1531198	16.5000000	11.7000000	3.2500000
.7388648	.6562286	.1531200	16.5000000	11.7000000	3.2500000
-.7388637	.6562276	.1531198	16.5000000	11.7000000	3.2500000

NOT A PLANAR PROBLEM

STRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS INTERSECT A LINE

2. PLANAR PROBLEM - TOO FEW SUPPORTS

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.0591533	.4850570	.6543264	1.3000000	25.2000000	6.4000000
.0915723	.9030155	.4197378	3.7500000	1.0400000	17.6300000

NOT A PLANAR PROBLEM

STRUCTURE UNSTABLE INSUFFICIENT NUMBER OF SUPPORTS

3. REDUNDANT STRUCTURE - MORE THAN THREE BARS

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS L	M	N	POINTS OF APPLICATION X	Y	Z
1.000000C	.0000000	.0000000	.0000000	.00000C0	.0000CC0
.0000000	1.000000C	.0000000	.000000C	.00C0CC0	.0000CC0
.0000000	.0000000	1.0000000	.0000000	.0000000	.0000CC0
1.0000000	.00C0000	.C000000	10.0000000	.0000000	.0000CC0
.0000000	1.0C00000	.0000000	10.0000000	.0000000	.00000C0
.0000000	.0000000	1.0000000	10.000000C	.0000CC0	.000CCC0

NOT A PLANAR PROBLEM

STRUCTURE INDETERMINATE EXCESS NUMBER OF SUPPORTS

4. UNSTABLE THREE BAR STRUCTURE - BARS ARE PARALLEL

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.0000000	1.0000000	.0000000	.0000000	.0000000	.0000000
.0000000	1.0000000	.0000000	10.0000000	.0000000	.0000000
.0000000	-1.0000000	.0000000	10.0000000	.0000000	.0000000

SUPPORT POINTS ARE ON A LINE
PLANAR PROBLEM

STRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS ARE PARALLEL

5. UNSTABLE THREE BAR STRUCTURE - BARS ARE NOT IN A PLANE

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.1000000	.9000000	.4242641	8.9000000	8.9000000	5.0000000
.9000000	.1000000	.4242641	8.9000000	8.9000000	5.0000000
.4246410	.1000000	.9000000	16.5000000	11.1000000	10.1000000

SUPPORT POINTS ARE ON A LINE
PLANAR PROBLEM

STRUCTURE UNSTABLE INSUFFICIENT NUMBER OF SUPPORTS

6. UNSTABLE THREE BAR STRUCTURE - LINES OF ACTION OF SUPPORTS INTERSECT

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
1.0000000	.0000000	.0000000	1.2000000	3.6000000	7.2000000
.0000000	1.0000000	.0000000	1.2000000	3.6000000	7.2000000
.4472136	.8944272	.0000000	3.2000000	7.6000000	7.2000000

SUPPORT POINTS ARE ON A LINE
PLANAR PROBLEM

STRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS ARE CONCURRENT

7. REDUNDANT STRUCTURE - MORE THAN THREE BARS

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
1.0000000	.0000000	.0000000	.0000000	.0000000	.0000000
1.0000000	.0000000	.0000000	10.0000000	.0000000	.0000000
1.0000000	.0000000	.0000000	10.0000000	.0000000	10.0000000
1.0000000	.0000000	.0000000	20.0000000	.0000000	10.0000000
1.0000000	.0000000	.0000000	10.0000000	.0000000	-10.0000000
1.0000000	.0000000	.0000000	20.0000000	.0000000	-10.0000000

SUPPORT POINTS ARE IN A PLANE
NOT A PLANAR PROBLEM

STRUCTURE INDETERMINATE EXCESS NUMBER OF SUPPORTS

8. UNSTABLE STRUCTURE - SUPPORTS ARE PARALLEL

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
-1.0000000	.0000000	.0000000	10.1000000	8.9000000	5.0000000
1.0000000	.0000000	.0000000	8.9000000	8.9000000	5.0000000
1.0000000	.0000000	.0000000	16.5000000	11.1000000	5.0000000

SUPPORT POINTS ARE IN A PLANE
PLANAR PROBLEM

STRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS ARE PARALLEL

9. UNSTABLE STRUCTURE - LINES OF ACTION OF SUPPORTS INTERSECT

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.0000000	1.0000000	.0000000	1.2000000	3.6000000	7.2000000
.4472136	.8944272	.0000000	3.2000000	7.6000000	7.2000000
1.0000000	.0000000	.0000000	3.6000000	3.6000000	7.2000000

SUPPORT POINTS ARE IN A PLANE
PLANAR PROBLEM

STRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS ARE CONCURRENT

10. UNSTABLE STRUCTURE - LESS THAN SIX BARS

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
1.0000000	.0000000	.0000000	6.3000000	8.3000000	.0000000
.0000000	.0000000	1.0000000	8.7000000	7.3000000	.0000000
.0000000	.0000000	1.0000000	5.2000000	8.5000000	.0000000
.0000000	1.0000000	.0000000	9.3000000	2.1000000	.0000000
.0000000	1.0000000	.0000000	5.4000000	3.2000000	.0000000

SUPPORT POINTS ARE IN A PLANE
NOT A PLANAR PROBLEM

STRUCTURE UNSTABLE INSUFFICIENT NUMBER OF SUPPORTS

11. REDUNDANT STRUCTURE - MORE THAN SIX BARS

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
-1.0000000	.00C0000	1.0000000	9.100000C	5.4000CC0	3.20000C0
1.0000000	.0000000	.0000000	6.3000000	8.3000000	.00000C0
1.0000000	.0000000	.0000000	8.7000000	7.3000000	.0000CC0
-1.0000000	.0000000	.0000000	9.1000000	6.4000000	.0000CC0
-1.0000000	.00C0000	.0000000	5.200000CC	8.5000CCC	.0000000
.0000000	1.0000000	.0000000	9.3000000	2.1000000	.00000C0
.0000000	1.0000000	.0000000	5.4000000	3.2000000	.0000CC0

NOT A PLANAR PROBLEM

STRUCTURE INDETERMINATE EXCESS NUMBER OF SUPPORTS

12. UNSTABLE STRUCTURE - SUPPORTS ARE PARALLEL

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.0000000	1.0000000	.0000000	.0000000	.0000000	.0000000
.0000000	-1.0000000	.0000000	10.0000000	.0000000	.0000000
.0000000	1.0000000	.0000000	.0000000	.0000000	10.0000000
.0000000	-1.0000000	.0000000	10.0000000	.0000000	10.0000000
.0000000	1.0000000	.0000000	.0000000	.0000000	-10.0000000
.0000000	-1.0000000	.0000000	10.0000000	.0000000	-10.0000000

SUPPORT POINTS ARE IN A PLANE
NOT A PLANAR PROBLEM

STRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS ARE PARALLEL

13. UNSTABLE STRUCTURE - SUPPORTS ARE IN PARALLEL PLANES

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
1.0000000	.0000000	.0000000	.0000000	.0000000	.0000000
.0000000	1.0000000	.0000000	.0000000	.0000000	.0000000
1.0000000	.0000000	.0000000	10.0000000	.0000000	10.0000000
.0000000	-1.0000000	.0000000	10.0000000	.0000000	10.0000000
.0000000	-1.0000000	.0000000	10.0000000	.0000000	-10.0000000
1.0000000	.0000000	.0000000	10.0000000	.0000000	-10.0000000

SUPPORT POINTS ARE IN A PLANE
NOT A PLANAR PROBLEM

STRUCTURE UNSTABLE SUPPORTS ARE IN PARALLEL PLANES

14. UNSTABLE STRUCTURE - FOUR OF SIX SUPPORTS ARE PARALLEL

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTICN COSINES OF BARS			POINTS OF APPLICATICN		
L	M	N	X	Y	Z
-1.0000000	.0000000	.0000000	9.1000000	6.4000000	.0000CC0
.0000000	1.00C0000	.0000000	6.3000000	8.3000000	.0000CC0
.0000000	.0000000	1.0000000	8.7000000	7.3000000	.0000CC0
.0000000	.00C0000	1.0000000	5.200000C	8.5000000	.0000CC0
.0000000	.0000000	-1.0000000	9.3000000	2.1000000	.0000CC0
.0000000	.0000000	1.0000000	5.4000000	3.2000000	.0000CC0

SUPPORT POINTS ARE IN A PLANE
NOT A PLANAR PROBLEM

STRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS INTERSECT A LINE

15. UNSTABLE STRUCTURE - LINES OF ACTION OF SUPPORTS INTERSECT

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTICN COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.4472136	.8944272	.0000000	3.2000000	7.6000000	7.2000000
.0000000	.8944272	-.4472136	1.2000000	7.6000000	5.2000000
-.4472136	.8944272	.0000000	-.8000000	7.6000000	7.2000000
.0000000	.8944272	.4472136	1.2000000	7.6000000	9.2000000
1.0000000	.0000000	.0000000	1.2000000	7.6000000	7.2000000
-1.0000000	.0000000	.0000000	1.2000000	7.6000000	7.2000000

SUPPORT POINTS ARE IN A PLANE
NOT A PLANAR PROBLEM

STRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS INTERSECT A LINE

16. UNSTABLE STRUCTURE - LESS THAN SIX BARS

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
1.0000000	.0000000	.0000000	8.7000000	7.3000000	.0000000
-.5801556	-.4850556	.6543245	1.3000000	25.2000000	6.4000000
.0915722	-.9030145	.4197373	3.7500000	1.0400000	17.6300000
.9000000	.4242641	-.1000000	11.6500000	12.9500000	16.3000000
.9000000	.1000000	.4242641	.0000000	.0000000	.0000000

NOT A PLANAR PROBLEM

STRUCTURE UNSTABLE INSUFFICIENT NUMBER OF SUPPORTS

17. REDUNDANT STRUCTURE - MORE THAN SIX BARS

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.4242641	.9000000	.1000000	.0000000	16.5000000	11.1000000
.0915722	-.9030145	.4197373	3.7500000	1.0400000	17.6300000
-.5801556	-.4850556	.6543245	1.3000000	25.2000000	6.4000000
1.0000000	.0000000	.0000000	.0000000	.0000000	.0000000
1.0000000	.0000000	.0000000	6.3000000	8.3000000	.0000000
-1.0000000	.0000000	.0000000	9.1000000	6.4000000	.0000000
.9000000	.1000000	.4242641	16.5000000	11.1000000	10.1000000

NOT A PLANAR PROBLEM

STRUCTURE INDETERMINATE EXCESS NUMBER OF SUPPORTS

18. UNSTABLE STRUCTURE - SUPPORTS ARE PARALLEL

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.0000000	1.0000000	.0000000	.0000000	4.1000000	.0000000
.0000000	1.0000000	.0000000	10.0000000	.0000000	.0000000
.0000000	1.0000000	.0000000	10.0000000	4.1000000	10.0000000
.0000000	1.0000000	.0000000	20.0000000	.0000000	10.0000000
.0000000	1.0000000	.0000000	10.0000000	.0000000	-10.0000000
.0000000	1.0000000	.0000000	20.0000000	.0000000	-10.0000000

NOT A PLANAR PROBLEM

STRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS ARE PARALLEL

19. UNSTABLE STRUCTURE - SUPPORTS ARE IN PARALLEL PLANES

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
1.0000000	.0000000	.0000000	1.6500000	2.9500000	6.3000000
1.0000000	.0000000	.0000000	12.0000000	29.0000000	.0000000
-1.0000000	.0000000	.0000000	12.0000000	29.0000000	.0000000
.1000000	.9000000	.4242641	.0000000	.0000000	.0000000
-1.0000000	.0000000	.0000000	8.7500000	.0000000	6.6600000
.1000000	.9000000	.4242641	8.7500000	.0000000	6.6600000

NOT A PLANAR PROBLEM

STRUCTURE UNSTABLE SUPPORTS ARE IN PARALLEL PLANES

20. UNSTABLE STRUCTURE - FOUR OF SIX SUPPORTS ARE PARALLEL

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS L	M	N	POINTS OF APPLICATION X	Y	Z
-1.0000000	.0000000	.0000000	.0000000	.0000000	21.3000000
1.0000000	.0000000	.0000000	.0000000	.0000000	21.3000000
-1.0000000	.0000000	.0000000	5.2000000	8.5000000	.0000000
.0000000	1.0000000	.0000000	9.3000000	2.1000000	.0000000
.0000000	.0000000	1.0000000	5.4000000	3.2000000	.0000000
1.0000000	.0000000	.0000000	.0000000	.0000000	.0000000

NOT A PLANAR PROBLEM

STRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS INTERSECT A LINE

21. UNSTABLE STRUCTURE - LINES OF ACTION OF SUPPORTS INTERSECT

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
1.0000000	.0000000	.0000000	1.2000000	3.6000000	7.2000000
.0000000	1.0000000	.0000000	1.2000000	3.6000000	7.2000000
.4472136	.8944272	.0000000	3.2000000	7.6000000	7.2000000
.0000000	.8944272	-.4472136	1.2000000	7.6000000	5.2000000
-.4472136	.8944272	.0000000	-.8000000	7.6000000	7.2000000
.0000000	.8944272	.4472136	1.2000000	7.6000000	9.2000000

NOT A PLANAR PROBLEM

STRUCTURE UNSTABLE LINES OF ACTION OF SUPPORTS INTERSECT A LINE

22a. PLANAR PROBLEM

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
1.0000000	.0000000	.0000000	21.2000000	.0000000	.0000000
1.0000000	.0000000	.0000000	35.3000000	21.2000000	.0000000
.0000000	-1.0000000	.0000000	35.3000000	21.2000000	.0000000

SUPPORT POINTS ARE ON A LINE
PLANAR PROBLEM

FIND THE RESULTANT TORSOR OF THE FOLLOWING LOADS

FORCES			APPLIED AT		
FX	FY	FZ	X	Y	Z
-692.17000	737.21000	.00000	16.10000	6.78000	.00000
17.02000	7092.22000	.00000	9.70000	5.95000	.00000
1021.12000	223.24000	.00000	3.40000	8.92000	.00000

COUPLES		
MX	MY	MZ
.0000000	.0000000	259.0000000
.0000000	.0000000	52.4900000
.0000000	.0000000	739.5000000

THE RESULTANT TORSOR IS,

```
F = (    345.9700000) I + (  8052.6699998) J + (         .0000000) K
M = (       .0000000) I + (       .0000000) J + ( 77956.8741989) K
```

THE BAR REACTIONS ARE

```
-9385.2741838  9731.2441885 -8052.6699998      .0000000      .0000000      .0000000
```

22b. PLANAR STRUCTURE WITH IMPROPER LOAD

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.1000000	.9000000	.4242641	1.3500000	7.6500000	19.7300000
-.1000000	.9000000	.4242641	1.3500000	7.6500000	19.7300000
.1000000	.9000000	.4242641	9.7500000	7.6500000	19.7300000

SUPPORT POINTS ARE ON A LINE
PLANAR PROBLEM

RESULTANT LOAD IS NOT IN PLANE OF STRUCTURE

23a. OBLIQUE PLANE PROBLEM RESULTANT FORCE IS IN PLANE

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.7388637	-.6562276	-.1531198	1.3000000	25.2000000	6.4000000
-.5801556	-.4850556	.6543245	16.5000000	11.7000000	3.2500000
-.0915722	.9030145	-.4197373	3.7500000	1.0400000	17.6300000

SUPPORT POINTS ARE IN A PLANE
PLANAR PROBLEM

FIND THE RESULTANT TORSOR OF THE FOLLOWING LOADS

FORCES			APPLIED AT		
FX	FY	FZ	X	Y	Z
-73.88637	65.62276	15.31198	1.30000	25.20000	6.40000
-58.01556	-48.50556	65.43245	16.50000	11.70000	3.25000
9.15722	-90.30145	41.97373	3.75000	1.04000	17.63000

THE RESULTANT TORSOR IS,

F = (-122.7447060) I + (-73.1842410) J + (122.7181560) K
M = (2524.7461075) I + (-1756.9239447) J + (1477.5325581) K

THE BAR REACTIONS ARE
-100.0000000 100.0000000 -100.0000000 .0000000 .0000000 .0000000

23b. OBLIQUE PLANE PROBLEM RESULTANT FORCE IS NOT IN PLANE

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
-.7388640	.6562279	.1531199	16.5000000	11.7000000	3.2500000
-.5801557	-.4850556	.6543246	3.7500000	1.0400000	17.6300000
.0915722	-.9030145	.4197373	1.3000000	25.2000000	6.4000000

SUPPORT POINTS ARE IN A PLANE
PLANAR PROBLEM

RESULTANT LOAD IS NOT IN PLANE OF STRUCTURE

23c. OBLIQUE PLANE PROBLEM RESULTANT FORCE IS NOT EXACTLY IN PLANE

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTICN COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.7388637	-.6562276	-.1531198	1.3000000	25.2000000	6.4000000
-.5801556	-.4850556	.6543245	16.5000000	11.7000000	3.2500000
-.0915722	.9030145	-.4197373	3.7500000	1.0400000	17.6300000

SUPPORT POINTS ARE IN A PLANE
PLANAR PROBLEM

RESULTANT LOAD IS NOT IN PLANE OF STRUCTURE

24. STABLE SPATIAL PROBLEM

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.1000000	.9000000	.4242641	.0000000	.0000000	.0000000
.9000000	.1000000	.4242641	.0000000	.0000000	.0000000
.9000000	.4242641	-.1000000	11.6500000	12.9500000	16.3000000
-.1000000	.9000000	-.4242641	11.6500000	12.9500000	16.3000000
.4242641	.1000000	.9000000	.0000000	9.7000000	7.9000000
.4242641	.9000000	.1000000	.0000000	9.7000000	7.9000000

SUPPORT POINTS ARE IN A PLANE
NOT A PLANAR PROBLEM

FIND THE RESULTANT TORSOR OF THE FOLLOWING LOADS

FORCES			APPLIED AT		
FX	FY	FZ	X	Y	Z
84.27500	-10.10000	19.24000	-6.50000	5.14000	1.50000

COUPLES		
MX	MY	MZ
78.3000000	68.5000000	87.1100000

THE RESULTANT TORSOR IS,

```
F = (    84.2750000) I + (   -10.1000000) J + (    19.2400000) K
M = (   192.3436000) I + (   319.9725000) J + (  -280.4135000) K
```

THE BAR REACTIONS ARE

```
    39.0014903    82.3859488    44.5109861   -28.6600889   -39.1484822   -47.3504682
```

25. STABLE SPATIAL PROBLEM

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.9000000	.1000000	.4242641	.0000000	11.6500000	.0000000
.1000000	.9000000	.4242641	11.6500000	.0000000	.0000000
-.1000000	.9000000	-.4242641	21.1200000	1.5000000	.0000000
.9000000	.4242641	-.1000000	.0000000	.0000000	16.3000000
.4242641	.1000000	.9000000	.0000000	9.7000000	7.9000000
.4242641	.9000000	.1000000	7.5000000	9.7000000	.0000000

NOT A PLANAR PROBLEM

FIND THE RESULTANT TORSOR OF THE FOLLOWING LOADS

FORCES			APPLIED AT		
FX	FY	FZ	X	Y	Z
30.12000	39.79000	10.20000	-8.70000	2.13000	1.00000

COUPLES		
MX	MY	MZ
19.9000000	900.1500000	10.2900000

THE RESULTANT TORSOR IS,

F = (30.1200000) I + (39.7900000) J + (10.2000000) K
M = (1.8360000) I + (1019.0100000) J + (-400.0386000) K

THE BAR REACTIONS ARE
-121.0548607 -106.1962450 -32.8508693 41.9365735 89.0738227 167.0425823

26. PLANAR PROBLEM

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

```
      DIRECTION COSINES OF BARS                         POINTS OF APPLICATION
    L               M               N               X               Y               Z
1.0000000        .0000000        .0000000      21.2000000        .0000000        .0000000
1.0000000        .0000000        .0000000      35.3000000      21.2000000        .0000000
 .0000000     -1.0000000        .0000000      35.3000000      21.2000000        .0000000
```

SUPPORT POINTS ARE ON A LINE
PLANAR PROBLEM

ALLOWABLE BAR LOADS

```
-9385.2741873  9731.2441878 -8052.6699986       .0000000       .0000000       .0000000
```

THE ALLOWABLE LOAD TORSOR IS

```
F = (    345.9700003) I + (  8052.6699986) J + (        .0000000) K
M = (       .0000000) I + (       .0000000) J + ( 77956.8741760) K
```

27. OBLIQUE PLANE PROBLEM RESULTANT FORCE IS IN PLANE

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.7388637	-.6562276	-.1531198	1.3000000	25.2000000	6.4000000
-.5801556	-.4850556	.6543245	16.5000000	11.7000000	3.2500000
-.0915722	.9030145	-.4197373	3.7500000	1.0400000	17.6300000

SUPPORT POINTS ARE IN A PLANE
PLANAR PROBLEM

ALLOWABLE BAR LOADS

-100.0000000	100.0000000	-100.0000000	.0000000	.0000000	.0000000

THE ALLOWABLE LOAD TORSOR IS
F = (-122.7447060) I + (-73.1842410) J + (122.7181560) K
M = (2524.7461074) I + (-1756.9239446) J + (1477.5325581) K

28. STABLE SPATIAL PROBLEM

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.1000000	.9000000	.4242641	.0000000	.0000000	.0000000
.9000000	.1000000	.4242641	.0000000	.0000000	.0000000
.9000000	.4242641	-.1000000	11.6500000	12.9500000	16.3000000
-.1000000	.9000000	-.4242641	11.6500000	12.9500000	16.3000000
.4242641	.1000000	.9000000	.0000000	9.7000000	7.9000000
.4242641	.9000000	.1000000	.0000000	9.7000000	7.9000000

SUPPORT POINTS ARE IN A PLANE
NOT A PLANAR PROBLEM

ALLOWABLE BAR LOADS

39.0014903 82.3859488 44.5109861 -28.6600889 -39.1484822 -47.3504682

THE ALLOWABLE LOAD TORSOR IS
F = (84.2750000) I + (-10.1000000) J + (19.2400000) K
M = (192.3436002) I + (319.9724995) J + (-280.4134998) K

29. STABLE SPATIAL PROBLEM

EXAMINE THE STABILITY OF THE FOLLOWING STRUCTURE

DIRECTION COSINES OF BARS			POINTS OF APPLICATION		
L	M	N	X	Y	Z
.9000000	.1000000	.4242641	.0000000	11.6500000	.0000000
.1000000	.9000000	.4242641	11.6500000	.0000000	.0000000
-.1000000	.9000000	-.4242641	21.1200000	1.5000000	.0000000
.9000000	.4242641	-.1000000	.0000000	.0000000	16.3000000
.4242641	.1000000	.9000000	.0000000	9.7000000	7.9000000
.4242641	.9000000	.1000000	7.5000000	9.7000000	.0000000

NOT A PLANAR PROBLEM

ALLOWABLE BAR LOADS

-121.0548607 -106.1962450 -32.8508693 41.9365735 89.0738227 167.0425823

THE ALLOWABLE LOAD TORSOR IS
F = (30.1200000) I + (39.7900000) J + (10.2000000) K
M = (1.8360000) I + (1019.0100006) J + (-400.0385996) K

APPENDIX L

Definition of "Zero"

The computer must convert any non-integer number to an approximate binary form which will fit its word length. The conversion and length limitation allow an accuracy of about ten decimal digits. With each calculation round-off errors accumulate. Whereas the flow charts ask if a quantity is zero, what must really be considered is whether the quantity is less than the maximum expected round-off error. If there are no conflicting requirements, the maximum round-off error may be estimated and, with perhaps some leeway, defined as "zero". However, where different orders of magnitude may be involved in the same problem or in subsequent problems or where it is advisable to have a large tolerance for one calculation and a small tolerance for another, the situation becomes far more complicated. One cannot define the "zeros" in terms of the number of significant figures in an expression; a specific magnitude is required.

An attempt was made to tie the size of the "zero" to the size of the structure. It was felt that barring unreasonable complications, such as having a very small structure at a great distance from the origin, the system of averaging the coordinates of the points of application and multiplying this average by the computer accuracy would give a suitable zero. However, instead of being

able to use 10^{-10} times the average coordinate, it was necessary to use a "zero" 10,000 times as big to get proper answers. Further investigation revealed that the difficulty probably involved the use of subroutine REAC1 in CONCURRENT1.

In subroutine REAC1 a small value of "zero" allows small diagonal elements which, during the back-solving process, generate large coordinates of the point of intersection. When these large values are then used in CONCURRENT1, the round-off error becomes much greater than the "zero" defined above. Unsuccessful efforts were made to modify the "zero" obtained above in a logical way for use in CONCURRENT1 and REAC1.

A multiplier of 10^{-6} gives proper answers, at least for this set of problems, but there is no other justification for its use. The author, regretfully, did not have sufficient time to pursue this investigation.